# Exceptional $\mathcal{N} = 6$ and $\mathcal{N} = 2$ $AdS_4$ supergravity, and zero-center modules



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# Exceptional $\mathcal{N} = 6$ and $\mathcal{N} = 2$ $AdS_4$ supergravity, and zero-center modules

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Abstract: We study the gauging of the orthosymplectic algebras OSp(6|4) × SO(2) and its "dual" OSp(2|4) × SO(6), both based on supergravities with the same exceptional coset SO*(12)/ U(6), and gauge group SO(6) × SO(2). The two dual theories are obtained by two different truncations of gauged $\mathcal{N} = 8$ $AdS_4$ supergravity. We explicitly study the gauge sector of the two dual theories with the most general group allowed by supersymmetry.

The $\mathcal{N} = 6$ gravity multiplet has also the exceptional property to be a *zero-center module* of OSp(6|4), as it is the case for superconformal Yang-Mills theory in four dimensions based on SU(2, 2|$n$) (PSU(2, 2|4) for $n = 4$) or OSp($n$|4).

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## Contents



## 1 Introduction

Gauged supergravities pertain to a topical subject of investigation because they are related to the possibility of turning on a scalar potential in an effective theory of gravity which can stabilize many of the scalar modes of the theory. Popular examples of such gaugings are those obtained by flux vacua in superstring theory [1].

Particular classes of these vacua can show residual supersymmetry both in Minkowski or anti de Sitter space, depending on the nature of the gauging of a given theory.

Minkowski vacua with residual supersymmetry correspond to theories with $\mathcal{N}$-extended Poincaré supersymmetry, with $0 \leq \mathcal{N} < 8$ at $D = 4$. Typical compactifications giving rise to such vacua are those based on *generalized Calabi-Yau manifolds* [2] or on *twisted tori* [3], the latter being the modern version of the gauging of *flat groups* à la Scherk-Schwarz [4]. These vacua give a realization of the so called no-scale models as they usually provide (partial) supersymmetry breaking with sliding gravitino mass and zero vacuum energy. In these compactifications one can then turn on further fluxes such as those giving rise to black holes and study interesting phenomena such as the *attractor mechanism* [5].

Another class of flux compactifications, whose interest is further motivated by additional physical properties, is the one corresponding to anti de Sitter vacua. These vacua



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are related to the famous $AdS_{d+1}/CFT_d$ correspondence, the most popular one being the $d = 4$ case [6, 7]. In this case the supergravity in question is the maximally extended gauged supergravity at $D = 5$ based on the superalgebra $\mathrm{SU}(2,2|4)$ [8]. Other examples of anti de Sitter supergravities relevant for the $AdS_{d+1}/CFT_d$ correspondence are those at $d = 3$ and $d = 6$, based on two different real forms of the orthosymplectic algebra $\mathrm{OSp}(8|4)$. However in recent times other classes of $AdS/CFT$ dual theories have been found, after realizing that superconformal invariant Chern-Simons theories can be constructed whose dual bulk supergravity theories correspond to lower $\mathcal{N}$ orthosymplectic algebras $\mathrm{OSp}(\mathcal{N}|4)$, with $2 \leq \mathcal{N} \leq 6$ [9–14].

It is the aim of the present paper to investigate some of the exceptional properties of the $\mathcal{N} = 6$ gauged supergravity theory and its "dual relation" to an $\mathcal{N} = 2$ theory based on the *exceptional model* related to $J_3^H$, one of the four degree-three Jordan algebras of the magic square introduced in [15–17]. Already in the ungauged case $\mathcal{N} = 6$ and $\mathcal{N} = 2$ supergravity, based on symmetric scalar manifold $\mathrm{SO}^*(12)/\,\mathrm{U}(6)$, exhibit a duality relation, since, although different in the fermionic sector, they have the same bosonic content. In particular they exhibit the same (large ) extremal black-hole attractor solutions where the role of the $BPS$ and non $BPS$ configurations in the two theories are exchanged. The superstring origin of these two ungauged theories was investigated in [18] in the context of compactifications on asymmetric orbifolds. Let us remark, moreover, that the duality between the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ four-dimensional theories has a three-dimensional counterpart in the duality between $\mathcal{N} = 12$ supergravity and the $\mathcal{N} = 4$ theory based on the exceptional quaternionic manifold $\mathrm{E}_{7(-5)}\,/[\mathrm{SU}(2) \times \mathrm{SO}(12)]$ [19, 20].

In the present investigation we concentrate on the gauging of these theories and we will show that both of these models can be obtained as truncations of the gauged $\mathcal{N} = 8$ theory of [21], with gauge structure $\mathrm{OSp}(6|4) \times \mathrm{SO}(2)$ in the $\mathcal{N} = 6$ case, and $\mathrm{OSp}(2|4) \times \mathrm{SO}(6)$ in the $\mathcal{N} = 2$ case. These superalgebras are indeed both subalgebras of the $\mathrm{OSp}(8|4)$ superalgebra, when one retains respectively 24 or 8 of the original 32 fermionic generators (anti de Sitter spinors). As far as the gauging is concerned, we analyze the consistency of the truncation procedure and, by use of the embedding tensor formalism, we give a detailed analysis of the gauge sector of both theories, for a generic group. We then work out the details in the particular case of the $\mathrm{SO}(2) \times \mathrm{SO}(6)$ gaugings, and determine the explicit form of the fermionic shifts and the scalar potential.

From a four-dimensional point of view, the $\mathcal{N} = 6$ theory is obtained just by gauging the $\mathrm{SO}(6)$ gauge group inside the U-duality group $\mathrm{SO}^*(12)$, which is the maximal group commuting with $\mathrm{SU}(2)$ inside $\mathrm{E}_{7(7)}$ (the U-duality group of $\mathcal{N} = 8$, $D = 4$ supergravity). On the other hand, the $\mathcal{N} = 2$ theory is obtained by gauging both $\mathrm{SO}(6) \subset \mathrm{SO}^*(12)$ and $\mathrm{SO}(2) \subset \mathrm{SU}(2)$, the global R-symmetry of the truncated $\mathcal{N} = 2$ theory, which does not participate to the gauging in the $\mathcal{N} = 6$ case.

Note that in the $\mathcal{N} = 6$ theory the spectrum is obtained from the $\mathcal{N} = 8$ spectrum by projecting out all $\mathrm{SU}(2)$ non-singlets. An extra $\mathrm{SO}(2)$ abelian symmetry, implied by the structure of the $\mathcal{N} = 6$ supergravity multiplet remains, commuting with the superalgebra. On the other hand in the $\mathcal{N} = 2$ theory the $\mathrm{SO}(6)$ symmetry commutes with the supercharges and it then merely acts as a matter flavor symmetry. It can therefore be gauged by the $\mathcal{N} = 2$ matter vectors, which precisely sit in the adjoint representation of $\mathrm{SO}(6)$. In fact, if in the truncation we would only keep SO(6) singlets, we would obtain pure $\mathcal{N} = 2$ anti de Sitter supergravity, with the gravitino mass induced by a $\mathcal{N} = 2$ Fayet-Iliopoulos term. This way to generate the gravitino mass is indeed common to all $\mathcal{N} = 2$ theories in $AdS_4$, when hypermultiplets are absent. The $\mathcal{N} = 2$ theory under investigation is a particular case of $AdS_4$ theories with superalgebras $\mathrm{OSp}(2/4) \times G_e$, where $G_e$ denotes the gauge symmetry, corresponding to the gauged isometries of the vector multiplets scalar manifold. Since the manifold spanned by the scalars in $\mathcal{N} = 2$ theory is only constrained by supersymmetry to be special-Kähler, it can be chosen to have any isometry $G$, so that we can always accommodate $G_e \subset G$ such that $G_e$ is an *electric subgroup* of $G$. The simplest example is the minimal series [22, 23] where $G = \mathrm{SU}(1, m)$ and $G_e$ can be embedded in $G$ for $m \geq \mathrm{Adj}\,[G_e]$. In this case the gauge symmetry is $\mathrm{SO}(2) \times G_e$, with an $AdS_4$ vacuum which is $\mathrm{SO}(2) \times G_e$ invariant. A similar embedding of $G_e$ in $G$ can be given also for $\mathcal{N} = 3$ supergravity with gauge group $\mathrm{SO}(3) \times G_e$ and for $\mathcal{N} = 5$ with gauge group SO(5). The latter theory was obtained long ago by de Wit and Nicolai [24].

Recently many of these theories have been shown to have a $CFT_3$ dual as a Chern-Simons gauge theory. In this case the $G_e$ commuting with $\mathrm{OSp}(\mathcal{N}|4)$ is identified with some flavor symmetry of the conformal theory matter multiplets. The $AdS_4$ theory with lowest supersymmetry is based on the $\mathrm{OSp}(1|4)$ superalgebra, with no gauge sector. In this case, symmetric $AdS_4$ vacua with any gauge group can be accommodated.

Note that the $\mathcal{N} = 6$ case has a special role among the $\mathcal{N}$-extended theories, because it is the only one with $\mathcal{N} > 4$ which contains an additional U(1) conserved current, and further because it is the only one which has a zero-center module supergravity multiplet, unlike $\mathcal{N} \neq 6$ orthosymplectic supergravities [25].

The paper is organized as follows: in section 2 we point out some exceptional properties of the $\mathrm{OSp}(6|4)$ algebra: to have the gravity multiplet as a *zero center module*, according to Flato and Fronsdal [25], and to have a zero Killing-Cartan form, which makes it more similar to the $\mathrm{SU}(2, 2|4)$ case than other orthosymplectic cases. The definition of such exceptional properties and of their possible physical implications is reported in this section. In section 4 we discuss the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ dual theories, both at the ungauged and gauged level, as they come from different truncations of $\mathcal{N} = 8$ (anti de Sitter or Poincaré) supergravity in four dimensions and we make some comments on the relation of $\mathcal{N} = 6$ supergravity with its ancestor theory, namely IIA supergravity compactified on $AdS_4 \times \mathbb{C}P_3$ [26–30], which is the higher dimensional theory underlying $\mathcal{N} = 6$ supergravity. In section 5 we briefly discuss a different $\mathcal{N} = 2$ truncation of the $\mathcal{N} = 8$ theory, in which the supergravity multiplet is coupled to 10 hypermultiplets and no vector multiplet. In appendix A examples of supergroups and supercosets with vanishing Killing-Cartan form are given. In appendix B the reader may find a list of branchings and decompositions which are used in our analysis. Finally in appendix C the spin-1/2 mass terms in the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ theories are given.

## 2 Zero center modules

The $\mathcal{N} = 6$ AdS-theory, besides being interesting in the light of some recent developments in string theory, also has some mathematical peculiarities, related to zero-center modules, that



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we want to elaborate on in the present section. Understanding their physical implications is beyond the scope of the present paper.

From a group-theoretical point of view, $\mathcal{N}=6$ supergravity on $AdS_4$ has two reasons for being exceptional: *1*) the superalgebra on which is based, namely OSp(6|4) has zero Killing-Cartan form and *2*) the zero-center module coincides with the supergravity multiplet.

Let us start recalling some basic facts about orthosymplectic superalgebras, the relation with supergravity backgrounds, the Killing-Cartan forms and the zero-center modules.

The compactification on $AdS_4 \times \mathbb{C}P_3$ of 10d type IIA string theory can be completely discussed in terms of the supermanifold

$$\frac{\mathrm{OSp}(6|4)}{\mathrm{U}(3)\times \mathrm{SO}(1,3)}\,. \tag{2.1}$$

The bosonic subgroup of the isometry group OSp(6|4) is SO(6) × Sp(4) and therefore the bosonic coset SO(6)×Sp(4)/ U(3)×SO(1,3) is the direct product of the homogeneous spaces $\mathbb{C}P^3 \times AdS_4$. In addition, there are fluxes associated to $F^{(4)} = g\epsilon$ and $F^{(2)} = k\mathcal{J}$ where $\epsilon$ and $\mathcal{J}$ are the Levi-Civita tensor in $AdS_4$ and the Kälher form on $\mathbb{C}P^3$, respectively. The fluxes $g$ and $k$ appear in the commutation relations of the supercharges. This background is a solution of type IIA supergravity in 10d (see [26–28, 30]). The fermionic sector is indeed described by 24 anticommuting supecharges $Q^A_\alpha$ in the fundamental representations of SO(6) and Sp(4). The superalgebra associated to (2.1) is given in terms of the bosonic generators $P_{\alpha\beta}$ (where $\alpha,\beta = 1,\ldots,4$ and are the Sp(4) generators) and $T^{AB}$ (where $A,B = 1,\ldots,6$ and they are SO(6) generators) and in terms of the fermionic generators $Q^A_\alpha$

$$\begin{aligned}
\{Q^A_\alpha, Q^B_\beta\} &= \eta^{AB} P_{\alpha\beta} + T^{AB}\epsilon_{\alpha\beta}\,,\\
[P_{\alpha\beta}, P_{\gamma\delta}] &= \frac{1}{2}\Big(\epsilon_{\gamma(\alpha}P_{\beta)\delta} + \epsilon_{\delta(\alpha}P_{\beta)\gamma}\Big)\,,\\
[T^{AB}, T^{CD}] &= \frac{1}{2}\Big(\eta^{C[A}T^{B]D} - \eta^{D[A}T^{B]C}\Big)\,,\\
[T^{AB}, Q^C_\alpha] &= \eta^{C[A}Q^{B]}_\alpha\,,\\
[P_{\alpha\beta}, Q^C_\gamma] &= \epsilon_{\gamma(\alpha}Q^C_{\beta)}\,.
\end{aligned} \tag{2.2}$$

In order to see the presence of the constants $g$ and $k$, we decompose the generators $P_{\alpha\beta} = \gamma^m_{\alpha\beta}P_m + g^{-1}\gamma^{mn}_{\alpha\beta}L_{mn}$ where $P_m$ are the generators of the coset and $L_{mn}$ are the SO(1,3) generators, and $T^{AB} = f^{AB}_{IJ}T^{IJ} + f^{AB}_{\bar I\bar J}T^{\bar I\bar J} + k^{-1} f^{AB}_{I\bar J}T^{I\bar J}$ where $T^{IJ}, T^{\bar I\bar J}$ are the generators of the coset SU(4)/ U(3) and $T^{I\bar J}$ are the generators of the subgroup. Therefore, when the algebra is decomposed into the generators of the subgroup U(3) × SO(1,3) one can see the two constants $g^{-1}, k^{-1}$ multiplying the generators of the subgroup. Accordingly, in the Maurer-Cartan equations of the coset, the coupling constants $g$ and $k$ multiply the $H$-connections.

The form of the superalgebra is the same for any $R$-symmetry group SO($\mathcal{N}$). The Killing-Cartan form for $\mathcal{N}=6$ vanishes. We have to recall that the Killing-Cartan form is defined as follows

$$K(X,Y) = \frac{1}{2}\mathrm{Str}\Big(\mathrm{ad}_X \mathrm{ad}_Y\Big), \tag{2.3}$$

where $X$, are generators of the supergroup and Str is the supertrace. In the appendix A, the supergroups with vanishing Killing-Cartan form are listed [31]. On the other hand the representations are classified according to the invariant tensors on the Lie superalgebra denoted Casimir operators. Given a non-degenerate Killing-Cartan metric there is a simple way to construct the basic quadratic Casimir. However, in general one can construct it as follows: consider the following restricted metric (which coincides with the Killing-Cartan form on the subgroups SO(6) and Sp(4) and on the supergenerators)

$$\langle P_{\alpha\beta}, P_{\gamma\delta}\rangle = \epsilon_{\alpha(\gamma}\epsilon_{\delta)\beta}\,, \qquad \langle T^{AB}, T^{CD}\rangle = \eta^{A[C}\eta^{D]B}\,, \qquad \langle Q^A_\alpha, Q^B_\beta\rangle = \epsilon_{\alpha\beta}\eta^{AB}\,, \tag{2.4}$$

where $\langle , \rangle$ denotes the trace, and define

$$C_2 = \epsilon^{\alpha\beta}\epsilon^{\gamma\delta}\,P_{\alpha\gamma}P_{\beta\delta} - \eta_{AC}\eta_{BD}\,T^{AB}T^{CD} + \epsilon^{\alpha\beta}\eta_{AB}\,Q^A_\alpha Q^B_\beta\,. \tag{2.5}$$

$C_2$ is constructed in terms of quadratic invariants of $\mathrm{Sp}(4) \times \mathrm{SO}(6)$. The coefficients of the linear combination OSp(6|4) invariant can be found by commuting $C_2$ with all the fermionic generators of the supergroup.[1] For OSp($\mathcal{N}$|4) there are other invariant Casimir operators that can be constructed with higher powers of generators.

The irreducible, positive energy representations of Sp(4) are fully characterized by the lowest value $E_0$ of the energy and by the spin $s$ and they are denoted by $D(E_0, s)$. The massless representations are $D(s+1, s)$ and the Dirac singleton are $D(1/2, 0)$ and $D(1, 1/2)$. Among the massless representations, $D(2, 1)$ has both Casimir operators equal to zero. (The same is also valid for the conformal group in 4d, namely SO(4, 2), whose representations $D(2, 1, 0)$ and $D(2, 0, 1)$ have vanishing Casimir operators). Those representations are referred to as *zero-center module* since the center of the enveloping algebra is zero. In analogy with the conformal group in 3d Sp(4) and with SO(4, 2), the zero-center module of a superalgebra is a representation characterized by the vanishing of all super-Casimir operators. A zero-center module is a special short representation of a superalgebra and it plays a role similar to the vacuum state.

According to [25], in the case of $AdS_4$ algebras one can find the following zero-center modules

$$\begin{aligned}
&\mathrm{OSp}(6|4)\ \mathcal{N}=6\ \ D(3,2|1)\oplus D(5/2,3/2|6)\oplus D(2,1|15)\oplus D(2,1|1)\\
&\qquad\qquad\qquad\quad \oplus D(3/2,1/2|20)\oplus D(3/2,1/2|\bar{6})\oplus D(1,0|15)\oplus D(1,0|\overline{15})\,,\\
&\mathrm{OSp}(5|4)\ \mathcal{N}=5\ \ D(5/2,3/2|1)\oplus D(2,1|5)\oplus D(2,1|1)\oplus D(3/2,1/2|10)\\
&\qquad\qquad\qquad\quad \oplus D(3/2,1/2|\bar{5})\oplus D(1,0|\overline{10})\oplus D(1,0|10)\\
&\mathrm{OSp}(4|4)\ \mathcal{N}=4\ \ D(2,1|1)\oplus D(3/2,1/2|4)\oplus D(1,0|6)\,,\\
&\mathrm{OSp}(3|4)\ \mathcal{N}=3\ \ D(2,1|1)\oplus D(3/2,1/2|3)\oplus D(3/2,1/2|1)\oplus D(1,0|6)\,,\\
&\mathrm{OSp}(2|4)\ \mathcal{N}=2\ \ D(2,1|1)\oplus D(3/2,1/2|2)\oplus D(1,0|2)\,.\\
&\mathrm{OSp}(1|4)\ \mathcal{N}=1\ \ D(2,1|1)\oplus D(3/2,1/2|1)\,.
\end{aligned} \tag{2.6}$$

where we have denoted by $D(s+1, s|n)$ respectively the Sp(4) representation and the dimension of the representation of the orthogonal group SO($\mathcal{N}$). Notice that only OSp(6|4)

[1] We would like to stress the analogy with abelian Lie algebras: the Killing-Cartan form is vanishing, but one can define an invariant bilinear form.

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has the supergravity multiplet (starting with the supergravity state $D(3,2|1)$ as the zero-center module (by the way, it is also the only supergroup of the $\mathrm{OSp}(\mathcal{N}|4)$ with vanishing Killing-Cartan form). For the supergroup OSp(5|4), the zero center module is represented by the gravitino multiplet $D(5/2, 3/2|1)$. The other four examples have, as zero-center module, the SYM multiplet with $\mathcal{N} = 1, 2, 3, 4$ supersymmetries. The technique to establish the existence of unitary zero-center module representations is that of the "induced representations" and it amounts to check if in the induced representation there is the trivial representation (the "vacuum"). In that case the module is a zero-center module.

Let us look to other series of supergroups with analogous peculiarities. As we can read from the appendix A there are other interesting supergroups with vanishing Killing-form which play an important role in superstring.[2] The most relevant one is the case of PSU(2, 2|4), with supercoset

$$\frac{\mathrm{PSU}(2,2|4)}{\mathrm{SO}(1,4)\times \mathrm{SO}(5)} \tag{2.7}$$

whose bosonic part is described by $AdS_5 \times S^5$. Again, one can study the sequence of supergroups $\mathrm{SU}(2,2|\mathcal{N})$ where $\mathcal{N} = 1, 2, 3$ and for each of them identifying the zero-center module. The fermionic sector is described by complex supercharges $Q^a_I, \overline{Q}^I_a$ (where $I$ is the SU(2, 2) index and $a = 1, \dots, \mathcal{N}$). However, we can observe the following fact: we can relate the supergroup PSU(2, 2|4) to the orthosymplectic OSp(4|4) by imposing the reality condition [32, 33]

$$Q^a_I = \epsilon_{IJ}\delta^{ab}\overline{Q}^J_b \,. \tag{2.8}$$

The invariant tensor $\epsilon_{IJ}$ breaks the group SU(2, 2) to SO(2, 3) $\sim$ Sp(4) while the invariant tensor $\delta^{ab}$ breaks the group $\mathrm{U}(n)$ down to $\mathrm{SO}(n)$. Therefore, we can relate the supergroup PSU(2, 2|4) with OSp(4|4) and the latter has the vector multiplet as zero-center module.

Another interesting example is the superalgebra SU(2, 2|3) which underlies the $N = 6$ supergravity on $AdS_5$ with gauge group U(3). It has a zero-center module which is the supersingleton of SU(2, 2|3). In the same way as above we can break SU(2, 2|3) down to OSp(3|4) which is the $\mathcal{N} = 3$ vector multiplet in $AdS_4$ (using the topological string model constructed on Grassmannian spaces (see [34, 35]) it should be possible to justify the selection rules discussed in [36]).

Notice that the OSp(3|4) has the vector representation as a zero-center module and therefore, one can argue that the zero-center module representation of OSp type are related to zero-center module representation of $SU$-type. To support this argument, we notice that the case OSp(1|4) which has the zero-center module which contains the vector multiplet, can be obtained by reducing it from SU(2, 2|1) which indeed has a zero-center module. Indeed, one can verify that the zero-center modules of $\mathrm{SU}(2,2|\mathcal{N})$ are mapped into zero-center modules of $\mathrm{OSp}(\mathcal{N}|4)$.

[2] Another example is OSp(4|2), which might play a role in non-critical strings. It has zero Killing-Cartan form and it would be interesting to study its zero-center modules.

## 3 Universal supergravity relations

We recall that in any supergravity theory there is a universal relation between the anti de Sitter cosmological constant and the gravitino mass. Indeed, for every four dimensional extended theory supersymmetry implies that the following Ward identity holds:

$$\delta^A_B\, V(\phi) = -12\, g^2\, \bar{S}^{AC} S_{CB} + g^2\, \bar{N}^A_I N^I_B \tag{3.1}$$

where $S_{AB}$ and $N^A_I$ are scalar field dependent matrices also appearing in the Lagrangian, the former defining the gravitino mass-like term:

$$2\, g\, S_{AB}\, \bar{\psi}^A_\mu\, \gamma^{\mu\nu}\, \psi^B_\nu\ + h.c.\,, \tag{3.2}$$

$2\, g\, S_{AB}$ being the gravitino mass matrix, the latter entering the spin-1/2 – gravitino couplings:

$$\mathrm{i}\, g\, \left( N^A_I\, \bar{\lambda}^I\, \gamma^\mu \psi_{\mu A}\ + h.c. \right)\,, \tag{3.3}$$

as reviewed in [37]. Here $A, B, \ldots$ are indices of the fundamental representation of the R-symmetry group $\mathrm{SU}(\mathcal{N}) \times \mathrm{U}(1)$, [3] their position (lower or upper) characterizing the left or right chirality of the gravitini, while the index $I$, enumerating the spin-1/2 fields, is a short-hand notation for the tensor character of the spin-1/2 fields.

The same matrices also appear in the order $g$ contribution to the supersymmetry transformation laws of the fermions which, as it is well known, is implied by the gauging procedure:

$$\delta\lambda_I = \cdots + g\, N^A_I\, \epsilon_A \tag{3.4}$$
$$\delta\psi_{\mu A} = \cdots + \mathrm{i}\, g\, S_{AB}\, \gamma_\mu\, \epsilon^B\,. \tag{3.5}$$

In an anti de Sitter background preserving all the $\mathcal{N}$ supersymmetries we have:

$$\delta\lambda_I = \cdots + g\, N^A_I\, \epsilon_A = 0 \qquad \Rightarrow N^A_I|_{\text{SuSy AdS}} = 0 \tag{3.6}$$
$$\delta\psi_{\mu A} = \cdots + \mathrm{i}\, g\, S_{AB}\, \gamma_\mu\, \epsilon^B \neq 0 \qquad \Rightarrow S_{AB}|_{\text{SuSy AdS}} \neq 0\,. \tag{3.7}$$

The precise relation, on the background, between the gravitino mass

$$m_{3/2} = 2\, g\, \sqrt{S_{AB} \bar{S}^{AB}/\mathcal{N}} \tag{3.8}$$

and the scalar potential is then found from eq. (3.1):

$$V(\phi|_{\text{SuSy AdS}}) = -3\, m^2_{3/2} = \Lambda \tag{3.9}$$

where $\Lambda$ is the cosmological constant.

Let us write down explicitly how the scalar potential specializes, following from the above relations, for the $\mathcal{N} = 2$ and $\mathcal{N} = 1$ cases, and what are the conditions to have an

[3] The U(1) factor being absent for the case $\mathcal{N} = 8$.



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anti de Sitter vacuum with unbroken gauge symmetry and preserving all supersymmetry. Note that the relations on the gauging of the $\mathcal{N}=1$ theory can also be obtained from the ones on $\mathcal{N}=2$-extended supergravity by a consistent truncation, as discussed in [38]. For the $\mathcal{N}=2$ theory, in the absence of hypermultiplets, we find [39]:[4]

$$V = -\frac{1}{2}(\Im\mathcal{N}^{-1})^{\Lambda\Sigma}\mathcal{P}_\Lambda\mathcal{P}_\Sigma + (U^{\Lambda\Sigma} - 3\bar{L}^\Lambda L^\Sigma)\mathcal{P}^x_\Lambda\mathcal{P}^x_\Sigma \tag{3.10}$$

where $\mathcal{P}_\Lambda$ is the prepotential for special geometry and $\mathcal{P}^x_\Lambda$ the constant quaternionic prepotential corresponding to a Fayet-Iliopoulos term. The first term in eq. (3.10) is usually written $g_{i\bar{\jmath}}\,k^i_\Lambda\,k^{\bar{\jmath}}_\Sigma\,\bar{L}^\Lambda L^\Sigma$, see for instance [39], where $k^i_\Lambda$ are the Killing vectors of the special Kähler manifold and $L^\Lambda$ is the upper part of the covariantly holomorphic symplectic section. Using $k^i_\Lambda = i\,g^{i\bar{\jmath}}\partial_{\bar{\jmath}}\mathcal{P}_\Lambda$, the orthogonality relations $\mathcal{P}_\Lambda L^\Lambda \equiv \mathcal{P}_\Lambda\bar{L}^\Lambda = 0$ and the definition $U^{\Lambda\Sigma} \equiv g^{i\bar{\jmath}} f^\Lambda_i \bar{f}^\Sigma_{\bar{\jmath}} = -\frac{1}{2}(\Im\mathcal{N}^{-1})^{\Lambda\Sigma} - \bar{L}^\Lambda L^\Sigma$, where $f^\Lambda_i = \mathcal{D}_i\,L^\Lambda$ and $\mathcal{N}_{\Lambda\Sigma}$ is the kinetic matrix of the vector fields, one easily retrieves the expression in (3.10) from the general one in [39]. The condition for an anti de Sitter supersymmetric background with unbroken gauge group is

$$\mathcal{P}_\Lambda|_{\rm vac} = 0\,, \qquad \left(U^{\Lambda\Sigma}\mathcal{P}^x_\Lambda\mathcal{P}^x_\Sigma\right)_{\rm vac} = 0 \qquad \mathcal{P}^x_\Lambda|_{\rm vac} \neq 0 \tag{3.11}$$

For the $\mathcal{N}=1$ case, instead, the scalar potential has the general form [40]:

$$V = e^{\mathcal{K}}\left[\mathcal{D}_i W \mathcal{D}_{\bar{\jmath}}\bar{W}\,g^{i\bar{\jmath}} - 3|W|^2\right] + \frac{1}{2}(\Im f^{-1})^{AB} D_A D_B \tag{3.12}$$

where $f_{AB}$ denotes the holomorphic vector kinetic matrix, $W(\phi)$ is the superpotential appearing in the fermion shifts of the chiral multiplet fermions and $D_A$ is the D-term appearing in the fermion shifts of the gaugini in the presence of gauged isometries in the chiral multiplet sector. In this case, the condition for an anti de Sitter vacuum preserving all supersymmetries and gauge symmetry is

$$D_A|_{\rm vac} = 0\,, \qquad \mathcal{D}_i W|_{\rm vac} = 0 \qquad W|_{\rm vac} \neq 0 \tag{3.13}$$

The cosmological constant is then, in this case

$$\Lambda = V|_{\rm vac} = -3\,\left(e^{\mathcal{K}}\,|W|^2\right)_{\rm vac} \tag{3.14}$$

and the gravitino mass is

$$m_{3/2} = \left(|W|\,e^{\mathcal{K}/2}\right)_{\rm vac} \tag{3.15}$$

[4] Here we use the conventions of [39] in which the imaginary part $\Im\mathcal{N}$ of the vector kinetic matrix $\mathcal{N}$ is negative definite.

## 4 Dual $\mathcal{N}=6$ and $\mathcal{N}=2$ gauged theories

It is known that ungauged $\mathcal{N}=6$ supergravity can be obtained from ungauged $\mathcal{N}=8$ supergravity by truncating out two gravitini multiplets. At a group theoretical level this corresponds to decomposing the relevant fermionic SU(8) representations with respect to $\mathrm{SU}(6)\times\mathrm{SU}(2)\times\mathrm{U}(1)$, under which the **8** branches as

$$\mathbf{8}\rightarrow(\mathbf{6},\mathbf{1})_{+\frac{1}{2}}+(\mathbf{1},\mathbf{2})_{-\frac{3}{2}}\,, \tag{4.1}$$

and keeping only the singlets under SU(2). In the following we shall use the indices $i,j,\dots=1,\dots,8$ to label the **8** representation, which split into indices $\alpha,\beta,\dots=1,2$ labelling the $(\mathbf{1},\mathbf{2})$ and $A,B,\dots=1,\dots,6$ labelling the $(\mathbf{6},\mathbf{1})$.[5] Equation (4.1) implies that the $\mathcal{N}=8$ gravitini $\psi^i_\mu$ decompose under $\mathrm{SU}(6)\times\mathrm{SU}(2)\times\mathrm{U}(1)\subset\mathrm{SU}(8)$, as $\psi_{i\,\mu}\rightarrow(\psi_{A\,\mu},\psi_{\alpha\,\mu})$, while the spin 1/2 fields $\chi_{ijk}$ into $(\chi_{ABC},\chi_{AB\alpha},\chi_{A\alpha\beta}\equiv\chi_A\epsilon_{\alpha\beta})$, according to the following branching

$$\mathbf{56}\rightarrow(\mathbf{20},\mathbf{1})_{+\frac{3}{2}}+(\mathbf{6},\mathbf{1})_{-\frac{5}{2}}+(\mathbf{15},\mathbf{2})_{-\frac{1}{2}}\,. \tag{4.2}$$

The **28** SU(8) representation of the $\mathcal{N}=8$ central charges $Z_{ij}$ branches in the following way

$$\mathbf{28}\rightarrow(\mathbf{15},\mathbf{1})_{+1}+(\mathbf{1},\mathbf{1})_{-3}+(\mathbf{6},\mathbf{2})_{-1}\,, \tag{4.3}$$

where $(\mathbf{15},\mathbf{1})_{+1}+(\mathbf{1},\mathbf{1})_{-3}$, to be labeled by the index $\underline{\Lambda}$, represent the $\mathcal{N}=6$ central charges $Z_{AB}$ and the singlet $Z_{\alpha\beta}=Z\,\epsilon_{\alpha\beta}$, while the remaining charges in the $(\mathbf{6},\mathbf{2})_{-1}$ are truncated.

The corresponding branching of the SU(8) representation **70** pertaining to the scalar fields $\phi^{ijkl}$, spanning $\mathscr{M}_{(\mathcal{N}=8)}=\mathrm{E}_{7(7)}\,/\,\mathrm{SU}(8)$, reads:

$$\mathbf{70}\rightarrow(\mathbf{15},\mathbf{1})_{-2}+(\overline{\mathbf{15}},\mathbf{1})_{+2}+(\mathbf{20},\mathbf{2})_{0}\,. \tag{4.4}$$

The truncation to the SU(2) singlets yields the 30 scalar fields of the $\mathcal{N}=6$ theory which span the coset manifold

$$\mathscr{M}_{(\mathcal{N}=6)}=\frac{\mathrm{SO}^*(12)}{\mathrm{U}(6)}\,, \tag{4.5}$$

which is a submanifold of $\mathscr{M}_{(\mathcal{N}=8)}$[6,7] of the theory is $\mathrm{SO}^*(12)$ which acts as a generalized electric-magnetic duality. The 32 electric-magnetic charges are indeed obtained by branching the $\mathrm{E}_{7(7)}$ representation **56** of the corresponding $\mathcal{N}=8$ charges with respect to the

[5] Here and in the following we reserve the indices $A,B,\dots$ only to label the fundamental representation of the U(6) R-symmetry group, while in the previous section they were associated with the fundamental representation of the R-symmetry group of a generic $\mathcal{N}$-extended supergravity.

[6] For the definition of $\mathrm{SO}^*(12)$, as a real form of $\mathrm{SO}(12,\mathbb{C})$, we refer the reader to standard group theory books [41]. See also [42].

[7] We recall that the subgroup $G_e$ of the on-shell global symmetry group of the theory, which transforms the electric field strengths into electric field strengths, is a global off-shell symmetry of the Lagrangian. In general $G_e$ depends on the symplectic frame (choice of the electric and magnetic charges out of the **32**). The ungauged Lagrangian that we consider, however, will be invariant with respect to the $G_e=\mathrm{SO}(6)$ subgroup of $\mathrm{SO}^*(12)$, which is the gauge group of the AdS theory. In this symplectic frame the vector fields $A^\Lambda_\mu$ are labeled by the index $\Lambda$ whose first value $\Lambda=0$ is chosen to correspond to the SO(6) singlet, while the remaining 15 values label the adjoint representation of SO(6).



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maximal subgroup $SO^*(12) \times SU(2)$ of $E_{7(7)}$ and keeping only the singlets:

$$\mathbf{56} \rightarrow (\mathbf{12}, \mathbf{2}) + (\mathbf{32}, \mathbf{1})\,. \tag{4.6}$$

We recall that the spinorial representation **32** of $SO^*(12)$ is real. If in the $\mathcal{N} = 8$ theory we truncate the multiplets of the six gravitini fields $\psi_{A\mu}$ instead, we would obtain an $\mathcal{N} = 2$ theory with the same bosonic sector as the $\mathcal{N} = 6$ model but a different fermionic field content. The $\mathcal{N} = 8$ central charges give now rise to the $\mathcal{N} = 2$ central charge $Z$ and 15 matter charges $Z_{AB}$. This theory therefore describes $\mathcal{N} = 2$ supergravity coupled to 15 vector multiplets and no hypermultiplets. The scalar fields in the vector multiplets span the special Kähler manifold (4.5). The spin 3/2 fields $\psi_{\alpha\mu}$ belong to the $(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$ representation in (4.1) while the spin 1/2 fields $\chi_{AB\alpha}$ are defined by the $(\mathbf{15}, \mathbf{2})_{-\frac{1}{2}}$ representation in the branching (4.2). This peculiarity of the $\mathcal{N} = 2$ and $\mathcal{N} = 6$ truncations just discussed, to share the same bosonic content although differing in the fermionic sector, was exploited in the study of extremal black holes, where one finds a class of common extremal solutions, which, however, have different supersymmetry properties in the two theories: the BPS solution of the $\mathcal{N} = 6$ theory is non-BPS in the $\mathcal{N} = 2$ one and vice versa [43–47].

To summarize the $\mathcal{N} = 8 \rightarrow \mathcal{N} = 6$, $\mathcal{N} = 2$ truncations discussed above, let us denote by $\Phi_{(in)}$ the (bosonic and fermionic) fields surviving the truncation and by $\Phi_{(out)}$ those fields which are truncated away. For the two truncations these fields read:

$$\mathcal{N} = 6 : \begin{cases} \Phi_{(in)} = \{\phi^{AB\alpha\beta} = \phi^{AB}\epsilon^{\alpha\beta}, A_\mu^{\alpha\beta},\, A_\mu^{AB},\, \psi_\mu^A,\, \chi^{ABC},\, \chi^A,\, c.c\} \\ \Phi_{(out)} = \{\phi^{ABC\beta},\, A_\mu^{A\alpha},\, \psi_\mu^\alpha,\, \chi^{AB\alpha},\, c.c.\} \end{cases}, \tag{4.7}$$

$$\mathcal{N} = 2 : \begin{cases} \Phi_{(in)} = \{\phi^{AB\alpha\beta} = \phi^{AB}\epsilon^{\alpha\beta}, A_\mu^{\alpha\beta},\, A_\mu^{AB},\, \psi_\mu^\alpha,\, \chi^{AB\alpha},\, c.c\} \\ \Phi_{(out)} = \{\phi^{ABC\beta},\, A_\mu^{A\alpha},\, \psi_\mu^A,\, \chi^{ABC},\, \chi^A,\, c.c.\} \end{cases} \tag{4.8}$$

Let us now consider the gauging of these $\mathcal{N} = 6$ and $\mathcal{N} = 2$ theories. As we shall show such gauged theories can all be constructed as a truncation of the $\mathcal{N} = 8$ theory with a suitable gauging. The most general $\mathcal{N} = 8$ gauged supergravity can be written in a manifestly SU(8) invariant form [21], in which the fermion shifts, which define the fermion mass terms and the scalar potential, consist in a symmetric tensor $S_{ij} = S_{ji}$ and a tensor $N^i{}_{jkl}$ in the **36** and **420** of SU(8) respectively.[8] In terms of these quantities, the supersymmetry variations of the (chiral components of the) fermion fields read:

$$\delta\psi_\mu^i = \cdots + i\, g\, S^{ij}\, \gamma_\mu\, \epsilon_j\,, \tag{4.9}$$

$$\delta\chi^{ijk} = \cdots + g\, N_l{}^{ijk}\, \epsilon^l\,. \tag{4.10}$$

According to the general form (3.1) of the Ward identity, the $\mathcal{N} = 8$ scalar potential reads:

$$V^{(\mathcal{N}=8)}(\phi) = g^2 \left( \frac{1}{48}\, N_i{}^{jkl}\, N^i{}_{jkl} - \frac{3}{2}\, S^{ij}\, S_{ij} \right)\,. \tag{4.11}$$

[8] It is useful here to define the correspondence between our notation and the one used in [48–50], to be distinguished by a prime from the quantities denoted here with the same symbol: $\gamma^\mu = i\,\gamma'^\mu$, $\psi_\mu^i = \frac{1}{\sqrt{2}}\,\psi'^{\,i}_\mu$, $\epsilon^i = \sqrt{2}\,\epsilon'^{\,i}$, $S^{ij} = -\frac{1}{\sqrt{2}}\,A_1^{ij}$, $N_\ell{}^{ijk} = -\sqrt{2}\,A_{2\ell}{}^{ijk}$.



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As far as the supersymmetry transformation rules are concerned, for the order $g$ sector involving the fermion shifts we find the decomposition:

$$\delta\psi^A_\mu = \cdots + i\, g\, \left(S^{AB}\,\gamma_\mu\,\epsilon_B + S^{A\beta}\,\gamma_\mu\,\epsilon_\beta\right)\,, \tag{4.12}$$

$$\delta\psi^\alpha_\mu = \cdots + i\, g\, \left(S^{\alpha B}\,\gamma_\mu\,\epsilon_B + S^{\alpha\beta}\,\gamma_\mu\,\epsilon_\beta\right)\,, \tag{4.13}$$

and

$$\delta\chi^{ABC} = \cdots + g\, \left(N_D{}^{ABC}\,\epsilon^D + N_\beta{}^{ABC}\,\epsilon^\beta\right)\,, \tag{4.14}$$

$$\delta\chi^{AB\alpha} = \cdots + g\, \left(N_D{}^{AB\alpha}\,\epsilon^D + N_\beta{}^{AB\alpha}\,\epsilon^\beta\right)\,, \tag{4.15}$$

$$\delta\chi^{A} = \cdots + g\, \left(N_B{}^{A}\,\epsilon^B + N_\beta{}^{A}\,\epsilon^\beta\right)\,. \tag{4.16}$$

The above fermion shifts correspond respectively to the branchings:

$$\mathbf{36} \rightarrow (\mathbf{21},\mathbf{1})_{+1} + (\mathbf{6},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{3})_{-3} \tag{4.17}$$

$$\mathbf{420} \rightarrow (\mathbf{105},\mathbf{1})_{+1} + (\mathbf{20},\mathbf{2})_{+3} + (\mathbf{84},\mathbf{2})_{-1} + (\mathbf{15},\mathbf{1})_{+1} + (\mathbf{15},\mathbf{3})_{+1} + (\mathbf{35},\mathbf{1})_{-3} + (\mathbf{6},\mathbf{2})_{-1}\,. \tag{4.18}$$

In order to have a consistent truncation, the solutions of the equations of motion of the reduced theory must also be solution in the parent theory, namely

$$\frac{\delta\mathcal{L}}{\delta\Phi_{(out)}} \approx 0\,, \tag{4.19}$$

where $\approx 0$ have to be intended in a weak sense, namely at $\Phi_{(out)} \equiv 0$. This fact in particular implies that all terms in the Lagrangian bilinear in the fermions and containing one retained and one truncated fermion, must disappear in the reduction, otherwise the corresponding field equations obtained by varying the Lagrangian with respect to the truncated fermions, would not be (weakly) satisfied. Let us consider the following order $g$ fermion bilinears in the gauged $\mathcal{N} = 8$ Lagrangian, which can be derived from the general expression for the fermion mass-like terms (3.2) and (3.3):[9]

$$g\,\Bigg(4\,S_{A\alpha}\,\bar\psi^A_\mu\,\gamma^{\mu\nu}\psi^\alpha_\nu + \frac{1}{6}\,N^\alpha{}_{BCD}\,\bar\chi^{BCD}\,\gamma^\mu\psi_{\alpha\,\mu} + \\ + \frac{1}{2}\,N^A{}_{BC\alpha}\,\bar\chi^{BC\alpha}\,\gamma^\mu\psi_{A\,\mu} + N^\alpha{}_A\,\bar\chi^A\,\gamma^\mu\psi_{\alpha\,\mu}\Bigg) + h.c.\,, \tag{4.20}$$

Since the terms in eq. (4.20) are linear in the truncated fermions, we conclude that consistency of the two truncations requires the components of the $\mathcal{N} = 8$ fermion shifts which transform as doublets of SU(2), namely $S_{\alpha B}$ in the $(\mathbf{6},\mathbf{2})_{-1}$, $N^\beta{}_{ABC}$ in the $(\mathbf{20},\mathbf{2})_{+3}$, $N^D{}_{AB\alpha}$ in the $(\mathbf{84},\mathbf{2})_{-1}$ and $N^\beta{}_A$ in the $(\mathbf{6},\mathbf{2})_{-1}$, to be weakly zero. Therefore in order for the truncation of gauged $\mathcal{N} = 8$ to $\mathcal{N} = 6$ or $\mathcal{N} = 2$ to be consistent, the gauging

[9] Here we restrict to the $\psi\psi$ and $\psi\chi$ terms in the Lagrangian. We refer the reader to appendix C for the explicit form of the $\chi\chi$ mass-like terms.

must be such that, when restricted to the common scalar sector of the two truncations, the components of the fermion shifts transforming as doublets under SU(2) must vanish. From now on we shall assume this to be the case. The implications of this condition on the possible gauge groups will be discussed in the next subsection. The resulting $\mathcal{N}=6$ and $\mathcal{N}=2$ theories then involve the transformation rules:

$$\delta\psi^A_\mu = \cdots + i\, g\, S^{AB}\,\gamma_\mu\,\epsilon_B\,, \tag{4.21}$$

$$\delta\chi^{ABC} = \cdots + g\, N_D{}^{ABC}\,\epsilon^D\,, \tag{4.22}$$

$$\delta\chi^{A} = \cdots + g\, N_B{}^{A}\,\epsilon^B\,, \tag{4.23}$$

for the $\mathcal{N}=6$ theory, while for the $\mathcal{N}=2$ theory we have:

$$\delta\psi^\alpha_\mu = \cdots + i\, g\, S^{\alpha\beta}\,\gamma_\mu\,\epsilon_\beta\,, \tag{4.24}$$

$$\delta\chi^{AB\alpha} = \cdots + g\, N_\beta{}^{\alpha AB}\,\epsilon^\beta\,. \tag{4.25}$$

While $S_{AB}$, $S_{\alpha\beta}$, $N_B{}^A \equiv N_B{}^{A\alpha\beta}\,\epsilon_{\alpha\beta}/2$ are irreducible $\mathrm{SU}(6)\times\mathrm{SU}(2)\times\mathrm{U}(1)$-tensors in the $(\mathbf{21},\mathbf{1})_{+1}$, $(\mathbf{1},\mathbf{3})_{-3}$ and $(\mathbf{35},\mathbf{1})_{+3}$ respectively, the shift tensors $N_D{}^{ABC}$, $N_\beta{}^{\alpha AB}$ transform in reducible representations and can therefore be written as follows:

$$\begin{aligned} N_D{}^{ABC} &= \overset{\circ}{N}_D{}^{ABC} + \frac{3}{4}\,\delta^{[A}_D\,N^{BC]}\,,\\ N_\beta{}^{\alpha AB} &= \overset{\circ}{N}_\beta{}^{\alpha AB} - \frac{1}{2}\,\delta^\alpha_\beta\,N^{AB}\,, \end{aligned} \tag{4.26}$$

where the irreducible tensors $\overset{\circ}{N}_D{}^{ABC}$, $N^{AB}$, $\overset{\circ}{N}_\beta{}^{\alpha AB}$ transform in the $(\overline{\mathbf{105}},\mathbf{1})_{-1}$, $(\overline{\mathbf{15}},\mathbf{1})_{-1}$ and $(\overline{\mathbf{15}},\mathbf{3})_{-1}$ representations respectively (this implies in particular the properties $\overset{\circ}{N}_D{}^{ABC} = \overset{\circ}{N}_D{}^{[ABC]}$, $\overset{\circ}{N}_C{}^{ABC}=0$).

Let us note, however, that the shifts involved in the transformation of projected out gravitini and dilatini with respect to projected out supersymmetry parameters are in general different from zero. In the truncation to $\mathcal{N}=6$ they are $S^{\alpha\beta}$ and $N_\beta{}^{\alpha AB}$, while in the truncation to $\mathcal{N}=2$ they are $S^{AB}$, $N_D{}^{ABC}$ and $N_B{}^{A\alpha\beta} = N_B{}^A\,\epsilon^{\alpha\beta}$. Some of them still enter the Lagrangian, as it is the case for the $\overset{\circ}{N}_D{}^{ABC}$ component of $N_D{}^{ABC}$, which enters the fermion mass term in the $\mathcal{N}=2$ theory, or the $N^{AB}$ component of $N_\beta{}^{\alpha AB}$ which enters both in the shift tensor $N_D{}^{ABC}$ and in the spin-1/2 mass terms of the $\mathcal{N}=6$ truncation (see appendix C). These shifts moreover play a role in rewriting the $\mathcal{N}=8$ scalar potential in terms of the only fermion shifts pertaining to the two truncations. The simplest way to achieve this is perhaps to restrict the $\mathcal{N}=8$ Ward identity:

$$\delta^i_j V^{(\mathcal{N}=8)} = g^2\left(-12\,S^{ik}\,S_{jk} + \frac{1}{6}\,N_j{}^{k\ell m}N^i{}_{k\ell m}\right)\,, \tag{4.27}$$

to the $\mathcal{N}=6$ and $\mathcal{N}=2$ indices and to the common scalar content of the two truncations:

$$\delta^A_B\,V^{(\mathcal{N}=8)} \approx g^2\left(-12\,S^{AC}\,S_{BC} + \frac{1}{6}\,N_B{}^{CDE}N^A{}_{CDE} + N_B{}^C N^A{}_C\right)\,, \tag{4.28}$$

$$\delta^\alpha_\beta\,V^{(\mathcal{N}=8)} \approx g^2\left(-12\,S^{\alpha\gamma}\,S_{\beta\gamma} + \frac{1}{2}\,N_\beta{}^{\gamma AB}N^\alpha{}_{\gamma AB}\right)\,, \tag{4.29}$$

where, as usual $\approx$ denotes the restriction to $\Phi_{(in)}$. By tracing the above identities we obtain the scalar potential written in terms of $\mathcal{N}=6$ and $\mathcal{N}=2$ quantities respectively:

$$V^{(\mathcal{N}=8)} \approx V^{(\mathcal{N}=6)} = g^2 \left(-2\, S^{AB}\, S_{AB} + \frac{1}{36}\, N_A{}^{BCD} N^A{}_{BCD} + \frac{1}{6}\, N_A{}^B N^A{}_B\right), \tag{4.30}$$

$$V^{(\mathcal{N}=8)} \approx V^{(\mathcal{N}=2)} = g^2 \left(-6\, S^{\alpha\beta}\, S_{\alpha\beta} + \frac{1}{4}\, N_\alpha{}^{\beta AB} N^\alpha{}_{\beta AB}\right). \tag{4.31}$$

Note that the two expressions (4.30), (4.31) are alternative descriptions of a same functional, which is the restricted $\mathcal{N}=8$ potential. We conclude that the $\mathcal{N}=8$ Ward identity implies a non trivial relation between the $\mathcal{N}=6$ and $\mathcal{N}=2$ fermion shifts, which is crucial in order to rewrite the same restricted $\mathcal{N}=8$ potential in terms of the quantities pertaining to the two truncations.

### 4.1 The gaugings of the $\mathcal{N}=6$ and $\mathcal{N}=2$ truncations

Having discussed the general form of the $\mathcal{N}=6$ and $\mathcal{N}=2$ truncations of the (gauged) $\mathcal{N}=8$ theory, let us show that these describe respectively the most general gauged $\mathcal{N}=6$ theory and the most general $\mathcal{N}=2$ gauged supergravity, based on the scalar manifold (4.5). In other words we consider here the problem of characterizing the most general local symmetries which these models may exhibit. To this end it is useful to describe their gauging by using the *embedding tensor* formalism [48–50] (for recent reviews on the embedding tensor formalism and its application to flux compactifications see [51]). Let us briefly recall the main facts about this technique and consider the gauging of an extended supergravity with $n_v$ vector fields $A^\Lambda_\mu$, $\Lambda = 0,\ldots,n_v-1$, and a scalar manifold of the form $G/H$, where $G$ represents the on-shell (classical) global symmetry group and $H$ its maximal compact subgroup. The gauging procedure consists in promoting a suitable subgroup $\mathcal{G}$ of the global symmetry group of the Lagrangian to local symmetry, gauged by (a subset of) the electric potentials of the theory. The formalism introduced in [49, 50] allows to freely choose the candidate gauge group inside the full on-shell global symmetry group $G$ of the ungauged theory by allowing the minimal couplings to involve not just the electric fields but also the magnetic ones $A_{\Lambda\,\mu}$ in a symplectic covariant fashion.[10] In this way the analysis of all possible gaugings is no longer constrained by the choice of the original ungauged Lagrangian and can refer to the full non-perturbative symmetries of the ungauged theory. Let us use the index $M$ to label the symplectic representation $\mathbf{R}$ of $G$ in which the electric and magnetic charges transform, so that a generic symplectic vector reads $V^M = (V^\Lambda,\, V_\Lambda)$. We shall also denote by $\Omega_{MN}$ the symplectic invariant matrix. Finally let the index $n$ label the adjoint representation of $G$. The choice of the gauge algebra inside the Lie algebra of $G$, to be gauged by a subset of the electric and magnetic potentials, can be parametrized by a $G$-covariant embedding tensor $\theta_M{}^n$, which expresses the gauge generators $X_M$ as a linear combination of the generators $t_n$ of $G$: $X_M = \theta_M{}^n\, t_n$. By definition $\theta_M{}^n$ naturally belongs to the product $\mathbf{R}\times\mathbf{Adj}(G)$. The deformations of the original ungauged Lagrangian which

[10] Consistency of the construction also requires the addition of antisymmetric tensor fields in the adjoint representation of $G$. Additional gauge symmetries guarantee that the introduction of these extra fields does not add new degrees of freedom to the theory.



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yield the gauged one with the same amount of supersymmetries, can be written in terms of the embedding tensor in a $G$-invariant way. Consequently the gauged equations of motion and Bianchi identities formally exhibit the same global symmetries as the ungauged ones provided $\theta_M{}^n$ is transformed under $G$ as well. This action of $G$ extended to $\theta_M{}^n$ can be interpreted as a mapping between different gauged supergravities. The electric-magnetic duality action of the generators $t_n$ of $G$ is represented by symplectic matrices $(t_n)_M{}^P$, which are meant to act on the vectors of electric and magnetic charges. We can then define the $G$-tensor $X_{MN}{}^P = \theta_M{}^n\,(t_n)_N{}^P$, in the same representation as $\theta_M{}^n$. For theories with $\mathcal{N} \leq 2$ not all generators of $G$ are associated with an electric-magnetic duality action (as it is the case for the quaternionic isometries in $\mathcal{N} = 2$ theories). These symmetries have $(t_n)_M{}^N = 0$ and thus do not contribute to $X_{MN}{}^P$. Consistency of the construction of a gauged extended supergravity requires $\theta_M{}^n$ to satisfy some $G$-covariant constraints consisting of a linear condition on $X_{MN}{}^P$:

$$X_{(MN}{}^L\,\Omega_{P)L} = 0\,, \tag{4.32}$$

and the following quadratic conditions

$$\theta_M{}^m\,\theta_N{}^n\,f_{mn}{}^p + X_{MN}{}^P\,\theta_P{}^p = 0\,, \tag{4.33}$$

$$\theta_M{}^m\theta_N{}^n\,\Omega^{MN} = 0\,, \tag{4.34}$$

where $f_{mn}{}^p$ are the structure constants of $G$: $[t_n,\, t_m] = f_{mn}{}^p\,t_p$. Equation (4.33) expresses the requirement that $\theta_M{}^n$ be a gauge invariant quantity and implies the closure of the gauge algebra $\mathfrak{g}$ inside the Lie algebra of $G$: $[X_M,\, X_N] = -X_{MN}{}^P\,X_P$. Equation (4.34) guarantees mutual locality between the electric and magnetic components of $\theta_M{}^n$. In supergravities with $\mathcal{N} > 2$ all $t_n$ have non trivial electric-magnetic duality action and it can be shown that (4.32) and (4.33) imply (4.34). The quadratic conditions (4.33), (4.34) on the structure constants of the gauge algebra imply the Ward identity (3.1) which is crucial for the supersymmetry of the gauged Lagrangian.

Note that the constraints (4.32), (4.33) and (4.34) are manifestly $G$-covariant. The linear one in particular amounts to a condition on $G$-representation of the embedding tensor in the decomposition of $\mathbf{R} \times \mathbf{Adj}(G)$. For instance in the maximal theory $G = \mathrm{E}_{7(7)}$, $H = \mathrm{SU}(8)$, $\mathbf{R} = \mathbf{56}$, $\mathbf{Adj}(G) = \mathbf{133}$ and (4.32) implies that $\theta_M{}^n$ belong to the $\mathbf{912}$ representation in the decomposition of $\mathbf{56} \times \mathbf{133}$.

As far as the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ truncations are concerned, in both cases the global symmetry group $G$ can be identified with the maximal subgroup $\mathrm{SO}^*(12) \times \mathrm{SU}(2)$ of $\mathrm{E}_{7(7)}$, with the only difference that in the former theory the SU(2) has a trivial action since all fields are singlets with respect to it, while this is not the case for the latter model. In the $\mathcal{N} = 2$ truncation the SU(2) factor is a global symmetry group whose generators $t_x$, $x = 1, 2, 3$, have a trivial electric-magnetic duality action: $(t_x)_M{}^N = 0$. As we shall see the gauging of this SU(2) group amounts to introducing a Fayet-Iliopuolos term.

In both the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ theories, $\mathbf{R} = (\mathbf{32}, \mathbf{1})$, $\mathbf{Adj}(G) = (\mathbf{66}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})$ and the decomposition of $\mathbf{R} \times \mathbf{Adj}(G)$ reads

$$(\mathbf{32}, \mathbf{1}) \times [(\mathbf{66}, \mathbf{1}) + (\mathbf{1}, \mathbf{3})] \rightarrow (\mathbf{32}, \mathbf{1}) + (\mathbf{1728}, \mathbf{1}) + (\mathbf{352}, \mathbf{1}) + (\mathbf{32}, \mathbf{3})\,. \tag{4.35}$$

The constraint (4.32) implies that the representations in the above decomposition which are in common with the three times symmetric product of the $(\mathbf{32},\mathbf{1})$ should vanish. Since

$$[(\mathbf{32},\mathbf{1})\times(\mathbf{32},\mathbf{1})\times(\mathbf{32},\mathbf{1})]_{sym.}\rightarrow(\mathbf{32},\mathbf{1})+(\mathbf{4224},\mathbf{1})+(\mathbf{1728},\mathbf{1})\,, \tag{4.36}$$

we conclude that in both theories the most general gaugings are defined by an embedding tensor in the following representations:

$$\theta_M{}^n\in(\mathbf{352},\mathbf{1})+(\mathbf{32},\mathbf{3})\,. \tag{4.37}$$

The gaugings parametrized by an embedding tensor $\theta_M{}^x$ in the $(\mathbf{32},\mathbf{3})$ representation involve the SU(2) generators and therefore have no effect in the $\mathcal{N}=6$ theory.[11] In the $\mathcal{N}=2$ theory instead they correspond to introducing an electric-magnetic F-I term, corresponding to constant electric and magnetic momentum maps $\mathcal{P}^x_M=(\mathcal{P}^x_\Lambda,\mathcal{P}^{x\Lambda})\equiv\theta_M{}^x$. Condition (4.33) in this case expresses the *equivariance* of the (constant) momentum maps:

$$\mathcal{P}_M{}^x\,\mathcal{P}_N{}^y\,\epsilon_{xy}{}^z+X_{MN}{}^P\mathcal{P}_P{}^z=0\,,\,. \tag{4.38}$$

Note that the representations (4.37) occur in the branching of the $\mathbf{912}$ of $\mathrm{E}_{7(7)}$ with respect to $\mathrm{SO}^*(12)\times\mathrm{SU}(2)$

$$\mathbf{912}\rightarrow(\mathbf{12},\mathbf{2})+(\mathbf{220},\mathbf{2})+(\mathbf{352},\mathbf{1})+(\mathbf{32},\mathbf{3})\,, \tag{4.39}$$

and are the only non-doublet representations. From this we conclude that *the most general $\mathcal{N}=6$ gauged supergravity can be obtained from the gauged $\mathcal{N}=8$ supergravity by truncating the fields and the embedding tensor to the non-doublet representations with respect to* SU(2). Let us illustrate the implications of the above discussion on the fermion shifts and scalar potential of the gauged $\mathcal{N}=6$ supergravity.

In a generic gauged extended supergravity, the fermion shifts, which belong to representations of $H$, are linear in the embedding tensor. In an extended supergravity based on a homogeneous symmetric scalar manifold, they are in fact expressed in terms of the so called T-tensor (originally introduced in [21] for the maximal supergravity), which is an $H$-covariant quantity, obtained by "boosting" $\theta_M{}^n$ by means of the scalar-dependent coset representative $\mathcal{V}(\Phi)$:

$$T(\Phi,\theta)_{\underline{M}}{}^{\underline{n}}=(\mathcal{V}^{-1}\star\theta)_{\underline{M}}{}^{\underline{n}}\equiv\mathcal{V}^{-1}{}_{\underline{M}}{}^{M}\,\mathcal{V}_n{}^{\underline{n}}\theta_M{}^n\,, \tag{4.40}$$

where $\mathcal{V}_M{}^{\underline{M}}$ and $\mathcal{V}_n{}^{\underline{n}}$ are the matrix representations of the coset representative in the $\mathbf{R}$ and $\mathbf{Adj}(G)$ representations of $G$, while the underlined indices are acted on by $H$ transformations. If the scalar fields $\Phi$ and $\theta_M{}^n$ are simultaneously transformed by means of a $G$ transformation $\mathbf{g}$, $T(\Phi,\theta)$ transforms under a corresponding $H$-compensating transformation depending on $\Phi$ and $\mathbf{g}$. In this sense $T(\Phi,\theta)$ is an $H$ covariant quantity, and thus can

[11] This however does not imply that it is vanishing. Indeed this component of the embedding tensor is related to the $(\mathbf{352},\mathbf{1})$ by the quadratic constraints (4.33) and (4.34). This is apparent from our discussion of the $\mathcal{N}=8$ Ward identity, which shows that the $\mathcal{N}=6$ and $\mathcal{N}=2$ expressions of the scalar potential, (4.30), (4.31), coincide on the common bosonic sector. The $(\mathbf{352},\mathbf{1})$ contributes to both expressions while the $(\mathbf{32},\mathbf{3})$ component contributes to the latter only.

be decomposed into irreducible $H-$ representations. These irreducible components comprise the fermion shift tensors. However $T(\Phi, \theta)$ can also be viewed as a $G$-tensor, since it is obtained by acting on the $G$-tensor $\theta$ by means of a $G$-transformation $\mathcal{V}(\Phi)$. This implies that $T(\Phi, \theta)$ satisfies the same linear and quadratic constraints as $\theta$ and thus, in particular, that it should belong to the same $G$-representation as $\theta$. The quadratic constraints on $T(\Phi, \theta)$, on the other hand, imply the Ward identity for the fermion shifts. Therefore the $H$- representations defining the fermion shift tensors should appear in the branching of the embedding tensor (or T-tensor) $G$-representation with respect to $H$. For instance, in the $\mathcal{N} = 8$ theory, the branching of the **912** with respect to SU(8) yields the SU(8)-representations pertaining to $S^{ij}$ and $N_l{}^{ijk}$:

$$\mathbf{912} \rightarrow \mathbf{36} + \mathbf{420} + \overline{\mathbf{36}} + \overline{\mathbf{420}}\,. \tag{4.41}$$

Similarly, for the $\mathcal{N} = 2$ and $\mathcal{N} = 6$ theories, branching the common embedding tensor representation (4.37) with respect to the compact symmetry group $\mathrm{SU}(6) \times \mathrm{SU}(2) \times \mathrm{U}(1)$ we find

$$\begin{aligned}(\mathbf{352}, \mathbf{1}) + (\mathbf{32}, \mathbf{3}) \rightarrow\ & (\mathbf{35}, \mathbf{1})_{+3} + (\mathbf{21} + \mathbf{15} + \mathbf{105}, \mathbf{1})_{+1} + (\overline{\mathbf{21}} + \overline{\mathbf{15}} + \overline{\mathbf{105}}, \mathbf{1})_{-1} + \\ & + (\overline{\mathbf{35}}, \mathbf{1})_{-3} + (\mathbf{1}, \mathbf{3})_{+3} + (\mathbf{15}, \mathbf{3})_{+1} + (\mathbf{1}, \mathbf{3})_{-3} + (\overline{\mathbf{15}}, \mathbf{3})_{-1}\,. \end{aligned} \tag{4.42}$$

The correspondence of the above representations with the fermion shifts introduced in (4.21)–(4.25) is:

$$\mathcal{N} = 6: \qquad (\mathbf{35}, \mathbf{1})_{+3} \equiv N_B{}^A\,, \qquad (\overline{\mathbf{21}} + \overline{\mathbf{105}} + \overline{\mathbf{15}}, \mathbf{1})_{-1} \equiv (S^{AB}, N_D{}^{ABC})\,, \tag{4.43}$$

$$\mathcal{N} = 2: \qquad (\mathbf{1}, \mathbf{3})_{+3} \equiv S^{\alpha\beta}\,, \qquad (\overline{\mathbf{15}}, \mathbf{1})_{-1} + (\overline{\mathbf{15}}, \mathbf{3})_{-1} \equiv N_\beta{}^{\alpha AB}\,. \tag{4.44}$$

In next section the $\mathrm{SO}(6)\times\mathrm{SO}(2)$ gauging of the $\mathcal{N} = 6$ theory will be discussed in detail. It is defined by choosing as non-vanishing component of the embedding tensor corresponding to the only SO(6)-singlet in the $(\mathbf{352}, \mathbf{1})$ representation. As explained in footnote 11, this choice implies that also a suitable component in the $(\mathbf{32}, \mathbf{3})$ (singlet with respect to $\mathrm{SO}(6) \times \mathrm{SO}(2)$) be different from zero. The latter however will not contribute to couplings in the gauged theory. Similarly in section 4.3 we shall consider the same gauging in the $\mathcal{N} = 2$ dual model. Also in this case either components $(\mathbf{352}, \mathbf{1})$ and $(\mathbf{32}, \mathbf{3})$ will contribute.

### 4.2 $\mathcal{N} = 6$ with $\mathrm{SO}(6) \times \mathrm{SO}(2)$ gauge group

We shall now discuss $\mathcal{N} = 6$ gaugings in some detail and focus on the theory with $\mathrm{SO}(6) \times \mathrm{SO}(2)$ local symmetry. The SO(2) factor, being contained in the SU(2) global symmetry, has a trivial action on the $\mathcal{N} = 6$ fields, so that the corresponding gauge potential $A^0_\mu$ is not minimally coupled and the gauge group is really only SO(6). Our choice of including SO(2) in description of the gauge group is just to emphasize the parallelism between this $\mathcal{N} = 6$ and the $\mathcal{N} = 2$ model to be discussed in the next section.

We start defining the relation between the fermion shifts and the embedding tensor. Let $\mathcal{V}_M{}^{\underline{M}}$ denote the coset representative of the scalar manifold (4.5):

$$\mathcal{V}_M{}^{\underline{M}} = \begin{pmatrix} \bar{h}_\Lambda{}^{\underline{\Delta}} & h_{\Lambda\underline{\Delta}} \\ \bar{f}^{\Lambda\underline{\Delta}} & f^\Lambda{}_{\underline{\Delta}} \end{pmatrix}\,, \tag{4.45}$$

where the underlined indices label the U(6) representations in which the self dual and anti-self dual field strengths transform, and the blocks $\mathbf{f} \equiv (f^\Lambda{}_{\underline{\Lambda}})$, $\bar{\mathbf{f}} \equiv (\bar{f}^{\Lambda\underline{\Lambda}})$, $\mathbf{h} \equiv (h_{\Lambda\underline{\Lambda}})$, $\bar{\mathbf{h}} \equiv (\bar{h}_\Lambda{}^{\underline{\Lambda}})$ satisfy the relations:

$$(\mathbf{f}\,\mathbf{f}^\dagger)^T = \mathbf{f}\,\mathbf{f}^\dagger\,, \qquad (\mathbf{h}\,\mathbf{h}^\dagger)^T = \mathbf{h}\,\mathbf{h}^\dagger\,, \qquad \mathbf{f}\,\mathbf{h}^\dagger - \bar{\mathbf{f}}\,\mathbf{h}^T = i\,\mathbb{1}\,, \tag{4.46}$$

$$\mathbf{f}^\dagger\,\mathbf{h} - \mathbf{h}^\dagger\,\mathbf{f} = -i\,\mathbb{1}\,, \qquad \mathbf{f}^T\,\mathbf{h} - \mathbf{h}^T\,\mathbf{f} = \mathbf{0}\,. \tag{4.47}$$

Using the above properties we can write the general expression of $\mathcal{V}^{-1}$:

$$\mathcal{V}^{-1}{}_{\underline{M}}{}^M = \begin{pmatrix} -i\,f^\Lambda{}_{\underline{\Lambda}} & i\,h_{\Lambda\underline{\Lambda}} \\ i\,\bar{f}^{\Lambda\underline{\Lambda}} & -i\,\bar{h}_\Lambda{}^{\underline{\Lambda}} \end{pmatrix}. \tag{4.48}$$

The basic quantity in terms of which the fermion shifts are expressed is the T-tensor, introduced in the previous section. Since in the $\mathcal{N}=6$ theory all the generators of $G$ have a non trivial duality action, the gauging is totally characterized by the generalized structure constants $X_{MN}{}^P$. It is then convenient here to use a slightly different definition of the T-tensor, with respect to eq. (4.40), and construct it by *dressing* $X_{MN}{}^P$ with the scalar fields by means of the coset representative:

$$T_{\underline{M},\underline{N}}{}^{\underline{P}} = [\mathcal{V}^{-1}\star X]_{\underline{M},\underline{N}}{}^{\underline{P}} \equiv \mathcal{V}^{-1}{}_{\underline{M}}{}^M\,\mathcal{V}^{-1}{}_{\underline{N}}{}^N\,\mathcal{V}_P{}^{\underline{P}}\,X_{MN}{}^P\,. \tag{4.49}$$

To write the fermion shifts in terms of the above quantity, we can use the corresponding $\mathcal{N}=8$ relations and reduce them to the $\mathcal{N}=6$ theory. In the maximal gauged supergravity the following relation holds:

$$T_{ij,kl}{}^{pq} = -\frac{1}{2\sqrt{2}}\,\delta^{[p}_{[k}\,N^{q]}{}_{l]ij} - \sqrt{2}\,\delta^{[p}_{[k}\,S_{l][i}\,\delta^{q]}_{j]}\,. \tag{4.50}$$

We then find:

$$\begin{aligned} N^A{}_B &= -2\sqrt{2}\,T_{\alpha\beta,\,BC}{}^{AC}\,, & N_{AB} &= -\frac{8}{3}\sqrt{2}\,T_{C[A,B]E}{}^{CE}\,, \\ N^A{}_{BCD} &= -2\sqrt{2}\,T_{[CD,B]E}{}^{AE} - \frac{1}{4}\,\delta^A_{[B}\,N_{CD]}\,, & S_{AB} &= \frac{\sqrt{2}}{5}\,T_{C(A,B)E}{}^{CE}\,. \end{aligned} \tag{4.51}$$

Let us now consider the gauging of $\mathcal{G}=\mathrm{SO}(6)$. Since the embedding tensor, by construction, defines the gauge structure constants, it is itself a gauge invariant quantity, as expressed by eq. (4.33). This allows to define the embedding tensor corresponding to a given gauge group $\mathcal{G}$ by considering the singlets in the branching of the embedding tensor $G$-representation with respect to $\mathcal{G}$. In particular the embedding tensor corresponding to $\mathcal{G}=\mathrm{SO}(6)$ must be defined by a singlet in the branching of (4.37) with respect to the SO(6) maximal subgroup of SU(6). This singlet arises only from the **21** and $\overline{\mathbf{21}}$ in the branching (4.42): $\mathbf{21}\to\mathbf{20}+\mathbf{1}$. The SO(6) generators are gauged by the electric potentials which transform in its adjoint representation, labeled by the antisymmetric couple $[IJ]$, $I,J=1,\dots,6$. The index $\Lambda$ splits under SO(6) into a label for the singlet and $[IJ]$, so that the only non vanishing components of $X_{MN}{}^P$ read:

$$X_{I_1J_1,I_2J_2}{}^{I_3J_3} = 4\,g\,\delta^{[I_3}_{[I_1}\,\delta_{J_1][I_2}\,\delta^{J_3]}_{J_2]}\,, \qquad X_{I_1J_1}{}^{I_3J_3}{}_{I_2J_2} = -X_{I_1J_1,I_2J_2}{}^{I_3J_3}\,. \tag{4.52}$$

The tensors $T_{\alpha\beta,AB}{}^{CD}$ and $T_{AB,CD}{}^{EF}$ have the following general expression:

$$T_{\alpha\beta,AB}{}^{CD} = \frac{g}{2} f^{I_1 J}{}_{\alpha\beta} \left(f^{JJ_1}{}_{AB}\, \bar{h}_{I_1 J_1}{}^{CD} + h_{I_1 J_1\, AB}\, \bar{f}^{JJ_1\, CD}\right) , \tag{4.53}$$

$$T_{EF,AB}{}^{CD} = \frac{g}{2} f^{I_1 J}{}_{EF} \left(f^{JJ_1}{}_{AB}\, \bar{h}_{I_1 J_1}{}^{CD} + h_{I_1 J_1\, AB}\, \bar{f}^{JJ_1\, CD}\right) . \tag{4.54}$$

It is useful at this point to use a U(6) covariant parametrization of the coset (4.5) in which the scalar fields are described by the tensors $\phi_{AB}$, $\overline{\phi}^{AB}$ in the $\mathbf{15}+\overline{\mathbf{15}}$. The coset representative will have the following general form:

$$\mathcal{V}_M{}^{\underline{M}} = \mathcal{A}^\dagger \exp\left[\begin{pmatrix} 0 & \mathbf{0}_{1\times 15} & 0 & \phi_{CD} \\ \mathbf{0}_{15\times 1} & \mathbf{0}_{15\times 15} & \phi_{AB} & \frac{1}{2}\,\bar{\phi}^{EF}\,\epsilon_{EFABCD} \\ 0 & \bar{\phi}^{CD} & 0 & \mathbf{0}_{1\times 15} \\ \bar{\phi}^{AB} & \frac{1}{2}\,\phi_{EF}\,\epsilon^{EFABCD} & \mathbf{0}_{15\times 1} & \mathbf{0}_{15\times 15}\end{pmatrix}\right] ,$$

$$\mathcal{A} = \frac{1}{\sqrt{2}}\begin{pmatrix} \mathbb{1} & i\,\mathbb{1} \\ \mathbb{1} & -i\,\mathbb{1}\end{pmatrix} . \tag{4.55}$$

A bosonic background characterized by the value of the scalar fields at the origin $\phi_{AB} \equiv 0$ describes a maximally supersymmetric (i.e. $\mathcal{N}=6$) $AdS_4$ background. Indeed, since we have switched on only a component of the embedding tensor in the $\mathbf{21}+\overline{\mathbf{21}}$, at the origin the T-tensor will lie in the same representations and thus have vanishing projections on the $\mathbf{35}+\mathbf{105}$, which are nothing but the spin 1/2 shift matrices $N^A{}_B$, $N^A{}_{BCD}$. On such background then the spin 1/2 fields have vanishing supersymmetry variation, while $S^{AB} = a_1\,\delta^{AB}$, where $|a_1| = 2$. Since $V_0 = V(\phi=0) = -48\,g^2$ one easily verifies that $-3\,m^2_{\frac{3}{2}} = -12\,g^2\,|a_1|^2 = V_0$, which is condition (3.9) for a maximally supersymmetric $AdS_4$ solution. Note that the unbroken symmetry in the vacuum is $\mathrm{OSp}(6/4)\times\mathrm{SO}(2)$, where the SO(2) is gauged by the singlet gauge field under which no field of the theory is charged, as it should be since SO(2) commutes with the supersymmetry generators.

Let us analyze the relation between this four dimensional vacuum solution and the ten dimensional $AdS_4\times\mathbb{C}P^3$ solution of Type IIA superstring. This higher dimensional background, as recalled in section 2, is characterized by a 4- and a 2-form flux $F_{\mu\nu\rho\sigma} = g\,\epsilon_{\mu\nu\rho\sigma}$, $F_{IJ} = k\,\mathcal{J}_{IJ}$, $\mathcal{J}_{IJ}$ being the Kähler form on $\mathbb{C}P^3$. The former is invariant under SO(6) while the choice of the latter breaks $SO(6)$ into U(3). We may choose indeed $\mathcal{J}_{IJ}$ to be the U(1) generator in SO(6) commuting with SU(3). The U(4)-invariant $AdS_4$ vacuum at the origin is likely to describe this compactification. In fact we may wonder if the flux $F_{IJ}$ enters this effective $\mathcal{N}=6$ theory as a v.e.v. of a U(3) invariant scalar field, thus defining a U(3)-invariant vacuum characterized by two distinct parameters: $g$, $k$. As we shall see this is not the case. In order to work out all the U(3) invariant vacua of the $\mathcal{N}=6$ supergravity with $SO(6)$ gauging it suffices to compute the fermion shifts and the scalar potential as a function of the only complex singlet $\phi^{sing.}_{AB} = \phi\,\delta^{IJ}_{AB}\,\mathcal{J}_{IJ}$. The fermion shift tensors read:

$$N^A{}_B = -a_2\,\frac{\phi}{\bar{\phi}}\,\delta^{AD}\,\mathcal{J}_{DB}\,, \qquad N^A{}_{BCD} = -3\,a_2\,\delta^{AE}\,\mathcal{J}_{E[B}\mathcal{J}_{CD]}\,, \qquad S^{AB} = a_1\,\delta^{AB}\,, \tag{4.56}$$

where

$$a_2 = \frac{1}{2}\, e^{-3\,|\phi|}\,(e^{4\,|\phi|}-1)\,\left[\frac{\bar{\phi}}{|\phi|}\,(e^{2\,|\phi|}+1)-(e^{2\,|\phi|}-1)\right], \tag{4.57}$$

$$a_1 = -\frac{1}{4}\, e^{-3\,|\phi|}\,\left[\frac{\bar{\phi}}{|\phi|}\,(e^{2\,|\phi|}+1)^3-(e^{2\,|\phi|}-1)^3\right]. \tag{4.58}$$

The scalar potential is:

$$V^{(\mathcal{N}=6)}(\phi,\bar{\phi}) = -48\,g^2\,\cosh(2\,|\phi|)\,. \tag{4.59}$$

From the above result it is clear that the only U(3) invariant vacuum of the gauged $\mathcal{N}=6$ supergravity coincides with the SO(6) invariant, maximally supersymmetric, $AdS_4$ background at the origin. In section 4.4 we shall show that this $\mathcal{N}=6$ $AdS_4$ theory does not describe the spontaneously broken phase of a gauged $\mathcal{N}=8$ theory, for any gauging. It can be obtained only as a consistent truncation of the SO(8)-gauged $\mathcal{N}=8$ theory. The same holds true for the $\mathcal{N}=2$ $AdS_4$ theory to be discussed in next section.

### 4.3 $\mathcal{N}=2$ gauging with $\mathrm{SO}(2)\times\mathrm{SO}(6)$ gauge group

As we have seen above, in the absence of hypermultiplets the $\mathcal{N}=2$ scalar potential has the general form

$$V^{(\mathcal{N}=2)} = -\frac{1}{2}(\Im\mathcal{N}^{-1})^{\Lambda\Sigma}\mathcal{P}_\Lambda\mathcal{P}_\Sigma+(U^{\Lambda\Sigma}-3\bar{L}^\Lambda L^\Sigma)\mathcal{P}^x_\Lambda\mathcal{P}^x_\Sigma\,. \tag{4.60}$$

$\mathcal{P}^x_\Lambda$ is a constant Fayet-Iliopoulos term that, in the gauging at hand, can be chosen as:

$$\mathcal{P}^x_0 = 4\,g\,\delta^x_1\,,\qquad \mathcal{P}^x_\Lambda = 0 \text{ for } \Lambda\neq 0\,, \tag{4.61}$$

corresponding to the gauging of the global $\mathrm{SO}(2)\subset\mathrm{SU}(2)$ symmetry.[12] The propotential $\mathcal{P}_\Lambda$, with $\mathcal{P}_{\Lambda=0}=0$ is instead responsible for the gauging of the vector multiplets isometries, along the $G_e=\mathrm{SO}(6)$ Lie algebra.

The $AdS_4$ supersymmetric vacuum corresponds to

$$\mathcal{P}_\Lambda|_{\mathrm{vac}} = 0 \quad \text{and} \quad U^{00} = 0|_{\mathrm{vac}}\,. \tag{4.62}$$

In the background (4.62) we then obtain

$$V|_{AdS_4} = -3\,m^2_{3/2}\,, \tag{4.63}$$

where $m_{3/2}=4\,g$.

The condition

$$U^{00} = 0 \tag{4.64}$$

[12] This choice corresponds to the $\mathrm{SO}(6)\times\mathrm{SO}(2)$ singlet in the $(\mathbf{32},\mathbf{3})$ component of the embedding tensor. A corresponding SO(6)-singlet in the $(\mathbf{352},\mathbf{1})$ component will have to be switched on in the embedding tensor because of the quadratic constraints.

which is a necessary condition to preserve supersymmetry, is equivalent to set $\mathcal{D}_a L^0 = f_a^0 = 0$ $(a = 1, \dots 15)$ on the vacuum. Note that (4.64) is a crucial condition for the gauging. It describes how the SO(2) factor is coupled to the gauge fields. For instance, if we would adopt instead a parametrization for the symplectic sections based on a cubic prepotential, then we would find $U^{00} = 3|L^0|^2$, which corresponds to a Minkowski vacuum (rather than anti de Sitter), with broken supersymmetry and flat directions for $\mathcal{P}_\Lambda = 0$. For a gauge group $G_e$, this would also give solutions with $G_e \to \mathrm{U}(1)^{\mathrm{rank}\, G_e}$ through the Higgs mechanism and would correspond to a no-scale $\mathcal{N} = 2$ supergravity. The standard cubic parametrization corresponds to a manifestly $\mathrm{SU}^*(6)$ invariant setting, since this is the parametrization which comes from dimensional reduction of $D = 5$ supergravity. The manifest compact symmetry in this case is USp(6) rather that U(6), so the coordinates corresponding to the Cartan decomposition are not special coordinates, which in this setting would correspond to the entry $f_{\underline{\mathbf{1}}}^{\Lambda} = L^\Lambda$ of the matrix (4.45). In the Cartan parametrization we have $X^{\mathbf{15}} = f_{\underline{\mathbf{1}}}^{\mathbf{15}}/f_{\underline{\mathbf{1}}}^{\mathbf{1}}$, and the SO(6) invariant part corresponds to $X^{\mathbf{15}} = 0$.

We note that the simplest $\mathcal{N} = 2$ theory which exhibits vacua with an unbroken $\mathrm{OSp}(2/4) \times G_e$ algebra are $\mathcal{N} = 2$ vector multiplets minimally coupled to supergravity [22]. In this case one can easily show that the condition (4.64) is satisfied in the $G_e$ unbroken phase. These models, together with their spontaneously broken phases were studied in [52]. We remark that the special Kähler geometry underlying minimal couplings correspond to the $\mathbb{C}P^n$ non-compact manifolds $\mathrm{SU}(1,n)/\mathrm{U}(n)$. These are the only symmetric special geometry which cannot be lifted to five dimensions.

### 4.4 $\mathcal{N} = 6$ and $\mathcal{N} = 2$ $AdS_4$ backgrounds from gauged $\mathcal{N} = 8$ theory

In this section we show that the U(4) gauged $\mathcal{N} = 2$ and $\mathcal{N} = 6$ theories (the latter describing the low energy dynamics of Type IIA superstring on a certain $AdS_4 \times \mathbb{C}P^3$ background) cannot be viewed as spontaneously broken phases of a gauged $\mathcal{N} = 8$ theory, they are instead consistent truncations of the maximal supergravity with SO(8) gauging. This implies that the deformation, discussed in [13, 26], which takes $AdS_4 \times S^7$ to the $\mathcal{N} = 6$ $AdS_4 \times \mathbb{C}P^3$ is not described by the v.e.v. of a zero-mode on $AdS_4$, i.e. of a scalar field in the maximal four dimensional model with gauging SO(8). This is consistent with the fact that the only U(4)-invariant vacuum found by Warner in the eighties [53] has $\mathcal{N} = 0$ and should correspond to the compactification of $D = 11$ supergravity on a "stretched seven sphere" discussed in [54]. Here we shall show, using a group theoretical argument, that no U(4)-invariant $\mathcal{N} = 6$ vacuum can be found in any gauged $\mathcal{N} = 8$ supergravity.

We start by noting that in the $\mathcal{N} = 8$ theory, with respect to the common SO(8) subgroup of the $\mathrm{SL}(8,\mathbb{R})$ and SU(8) symmetry groups, the $\mathbf{8}$ of SU(8) and the $\mathbf{8}$ of $\mathrm{SL}(8,\mathbb{R})$ correspond to the representations $\mathbf{8}_s$ and $\mathbf{8}_v$ respectively. The U(4) symmetry group of the $\mathcal{N} = 6$ $AdS_4 \times \mathbb{C}P^3$ solution, is embedded inside SO(8) in such a way that the following branchings hold:

$$\begin{aligned} \mathbf{8}_s &\to \mathbf{1}_{+1} + \mathbf{1}_{-1} + \mathbf{6}_0\,, \\ \mathbf{8}_v &\to \mathbf{4}_{+\frac{1}{2}} + \overline{\mathbf{4}}_{-\frac{1}{2}} \\ \mathbf{8}_c &\to \mathbf{4}_{-\frac{1}{2}} + \overline{\mathbf{4}}_{+\frac{1}{2}}\,. \end{aligned} \tag{4.65}$$

Consequently the corresponding symmetric tensor product representations $\mathbf{35}_s$, $\mathbf{35}_v$, $\mathbf{35}_c$ branch in the following way:

$$\begin{aligned}
\mathbf{35}_s &\to \mathbf{1}_{+2} + \mathbf{1}_0 + \mathbf{1}_{-2} + \mathbf{6}_{+1} + \mathbf{6}_{-1} + \mathbf{20}_0\,, \\
\mathbf{35}_v &\to \mathbf{10}_{+1} + \overline{\mathbf{10}}_{-1} + \mathbf{15}_0\,, \\
\mathbf{35}_c &\to \mathbf{10}_{-1} + \overline{\mathbf{10}}_{+1} + \mathbf{15}_0\,,
\end{aligned} \tag{4.66}$$

the 70 scalar fields transform in the $\mathbf{35}_v + \mathbf{35}_c$, which can be described as the self-dual and anti self-dual components of the 4-times antisymmetric tensor product of the $\mathbf{8}_s$, respectively. We know that the most general gauging of the $\mathcal{N} = 8$ theory is encoded in an embedding tensor transforming in the $\mathbf{912}$ of $\mathrm{E}_{7(7)}$. This representation describes not just the plain embedding tensor $\theta_M{}^n$ defining the gauge algebra, which encodes the coupling constants of the gauged theory, but also the T-tensor $T(\Phi, \theta)$ introduced in (4.40). Therefore if the maximal theory with gauge group $\mathcal{G}$ admits a vacuum at $\langle\Phi\rangle \equiv \Phi_0$ with symmetry group $\mathcal{G}' \subset \mathcal{G}$, the physical quantities on such vacuum (masses, couplings etc. . . ) must be defined in terms of the T-tensor evaluated on this solution, namely $T_0 = T(\Phi_0, \theta)$, which must be a $\mathcal{G}'$-singlet. Since $T(\Phi, \theta)$ belongs to the $\mathbf{912}$ representation, a $\mathcal{G}'$-invariant vacuum is described by a $\mathcal{G}'$-singlet $(T_0)$ in the $\mathbf{912}$ which provides the fermion shift tensors computed on the vacuum. Moreover such quantity is subject to the quadratic constraints, which amount to the Ward identity on the fermion shift tensors.

With respect to SU(8) the $\mathbf{912}$ branches in the $\mathbf{36} + \mathbf{420}$, corresponding to the shift tensors $S_{ij}$ and $N^i{}_{jkl}$ respectively, and the conjugate representations. With respect to SO(8) the $\mathbf{36}$ branches into $\mathbf{1}+\mathbf{35}_s$, while the $\mathbf{420}$ branches into $\mathbf{35}_v+\mathbf{35}_c+\mathbf{350}$. Therefore the branching of the $\mathbf{912}$ with respect to SO(8) reads:

$$\mathbf{912} \to 2 \times (\mathbf{1} + \mathbf{35}_s + \mathbf{35}_v + \mathbf{35}_c + \mathbf{350})\,. \tag{4.67}$$

For each representation within parentheses the two copies are mapped into one another by interchanging the electric with the magnetic vector fields in the ungauged theory (exchanging the role of electric and magnetic charges) [48], which has no effect on the physics of the resulting gauged model. By choosing once for all the symplectic frame of the ungauged theory (namely the 16 vector fields of the model out of the $\mathbf{32}$ of $\mathrm{SO}^*(12)$), we automatically single out one of the two copies and thus can focus only on the representations within parentheses in (4.67). The singlet define the SO(8) gauging of de Wit and Nicolai. We may wonder if the $\mathbf{912}$ contains any other singlet, besides this one, with respect to the U(4) symmetry of the $\mathcal{N} = 6$ background. Since the $\mathbf{350}$ does not contain any U(4) singlet, from (4.66) we conclude that the only other singlet $T_0$ is the one contained in the $\mathbf{35}_s$ and corresponds to a symmetric $8 \times 8$ matrix $S^{ij}$ of the form

$$S^{ij} = \mathrm{diag}(s,\, s,\, s',\, s',\, s',\, s',\, s',\, s')\,, \tag{4.68}$$

where $2\,s + 6\,s' = 0$ from the tracelessness condition. So far we have not considered the effect of the quadratic constraints on the T-tensor $T_0$, which imply the Ward identity for the fermion shifts. Let us show that a generic component of T-tensor in the $\mathbf{35}_s$ violates the Ward identity, and therefore does not survive the quadratic constraint. Consider a generic $T_0 \in \mathbf{1} + \mathbf{35}_s$. It can be expressed in terms of a symmetric matrix $S^{ij} = S^{ji}$. Since $T_0$ has no component in the **420**, it will yield a vanishing dilatino shift, $N_i{}^{jkl} = 0$, while the gravitino shift will be described by the matrix $S^{ij}$ itself. The Ward identity at the origin would read:

$$V^{(\mathcal{N}=8)}\,\delta_i^j \propto S_{ik}\,S^{kj}\,. \tag{4.69}$$

the only solution to the above identity is $S^{ij} \propto \delta^{ij}$ $(s = s')$ which corresponds to the SO(8) gauging $T_0 \in \mathbf{1}$, with no component in the $\mathbf{35}_s$.

As far as the $\mathcal{N} = 0$ U(4)-invariant $AdS_4$ studied in [26, 53, 54] is concerned, the above argument about the Ward identity does not apply. Indeed the SU(4) symmetry groups pertaining to the $\mathcal{N} = 0$ and $\mathcal{N} = 6$ vacua are embedded in inequivalent ways inside SO(8), see appendix B. With respect to the U(4) symmetry group of the $\mathcal{N} = 0$ vacuum the following branching holds:

$$\begin{aligned} \mathbf{8}_c &\rightarrow \mathbf{1}_{+1} + \mathbf{1}_{-1} + \mathbf{6}_0\,, \\ \mathbf{8}_s &\rightarrow \mathbf{4}_{+\frac{1}{2}} + \overline{\mathbf{4}}_{-\frac{1}{2}} \\ \mathbf{8}_v &\rightarrow \mathbf{4}_{-\frac{1}{2}} + \overline{\mathbf{4}}_{+\frac{1}{2}}\,. \end{aligned} \tag{4.70}$$

Now it is the $\mathbf{35}_c$ representation which contains the U(4)-singlet. A U(4) invariant T-tensor $T_0$ would then be a combination of the SO(8) singlet and the U(4) singlet in the $\mathbf{35}_c$: $T_0 \in \mathbf{1} + \mathbf{35}_c$. The Ward identity would now allow a component of $T_0$ inside $\mathbf{35}_c$, since $\mathbf{35}_c$ is contained inside the **420** of SU(8), and thus $T_0$ will yield $S^{ij} \propto \delta^{ij}$, $N^i{}_{jkl} \neq 0$. The singlet $T_0$ in the $\mathbf{35}_c$, as any element of the same representation, can be obtained by acting on the SO(8)-singlet embedding tensor, defining the SO(8) gauging, by means of the coset representative $\mathcal{V}$ parametrized by a suitable scalar field $\phi^{ijkl}$, since the scalar fields transform in the $\mathbf{35}_v + \mathbf{35}_c$. The v.e.v. of such scalar field provides the deformation which determines the SO(8) $\rightarrow$ U(4) spontaneous symmetry breaking and the supersymmetry breaking $\mathcal{N} = 8 \rightarrow \mathcal{N} = 0$.

## 5 An $\mathcal{N} = 2$ truncation of the $\mathcal{N} = 8$ theory with no vector multiplets and ten hypermultiplets

We can consider a different $\mathcal{N} = 2$ truncation of the maximal theory in four dimensions with with no vector multiplets and ten hypermultiplets. This is the maximal $\mathcal{N} = 2$ truncation of the $\mathcal{N} = 8$ theory with no vector multiplets. The scalar fields span the manifold:

$$\mathcal{M}^{(\mathcal{N}=2)} = \frac{\mathrm{E}_{6(+2)}}{\mathrm{SU}(2)\times\mathrm{SU}(6)}\,. \tag{5.1}$$

The global symmetry group of the theory is $G = \mathrm{U}(1)\times\mathrm{E}_{6(+2)}$, which is a maximal subgroup of $E_{7(7)}$. This theory can indeed be obtained as a truncation of the four dimensional maximal supergravity. Since the graviphoton is the only vector field of the model, we may only gauge one abelian isometry of the quaternionic manifold. Let us describe all possible gaugings by means of the embedding tensor. This tensor belongs to the product of the symplectic representation $\mathbf{R}$ of the electric and magnetic charges, labelled by $M = 1, 2$, and the adjoint representation of $G$. In this case we have:

$$\mathbf{R} = \mathbf{1}_{+3} + \mathbf{1}_{-3} \ , \quad \mathbf{Adj}(G) = \mathbf{1}_0 + \mathbf{78}_0 \, , \tag{5.2}$$

and therefore

$$\theta_M{}^n \in \mathbf{R} \times \mathbf{Adj}(G) = \mathbf{1}_{+3} + \mathbf{1}_{-3} + \mathbf{78}_{+3} + \mathbf{78}_{-3} \, . \tag{5.3}$$

The singlets $\mathbf{1}_{\pm 3}$ do not correspond to a viable gauging since they would correspond to gauging the global U(1) symmetry by means of the graviphoton which is charged itself under this U(1). Therefore we are left with

$$\theta_M{}^n \in \mathbf{78}_{+3} + \mathbf{78}_{-3} \, . \tag{5.4}$$

Notice that the above representations enter the branching of the $\mathrm{E}_{7(7)}$ embedding tensor representation with respect to $G$:

$$\mathbf{912} \rightarrow \mathbf{78}_{+3} + \mathbf{78}_{-3} + \mathbf{27}_{+1} + \overline{\mathbf{27}}_{-1} + \mathbf{351}_{+1} + \overline{\mathbf{351}}_{-1} \, . \tag{5.5}$$

The fermion fields consist in the gravitini $\psi^\alpha_\mu$, $\alpha = 1, 2$, and 20 hyperini $\zeta^{ABC}$, $A = 1, \ldots, 6$. The corresponding gauge contribution to the supersymmetry transformation laws read:

$$\begin{aligned} \delta\psi^\alpha_\mu &= \cdots + i\, g\, S^{\alpha\beta}\, \gamma_\mu\, \epsilon_\beta \, , \\ \delta\zeta^{ABC} &= \cdots + g\, N_\alpha{}^{ABC}\, \epsilon^\alpha \, . \end{aligned} \tag{5.6}$$

The shift tensors $S^{\alpha\beta}$ and $N_\alpha{}^{ABC}$ transform in the representations $(\mathbf{1}, \mathbf{3})$ and $(\mathbf{20}, \mathbf{2})$ of the $H = \mathrm{SU}(2) \times \mathrm{SU}(6)$ subgroup of $G$, respectively. These representations appear, together with their conjugate, in the branching of the embedding tensor representation with respect to $H$:

$$\mathbf{78} \rightarrow (\mathbf{1}, \mathbf{3}) + (\mathbf{20}, \mathbf{2}) + (\mathbf{35}, \mathbf{1}) \, , \tag{5.7}$$

the latter representation correspond to a quantity $N_A{}^B$ which does not appear in the theory as a fermion shift matrix, though it enters in the expression of the hyperino mass matrix:

$$M^{ABC,\, EFG} = -\frac{1}{24}\, \epsilon^{A_1 A_2 A_3 B [B_1 B_2}\, N^{B_3]}{}_B \, . \tag{5.8}$$

If we interpret this theory as a truncation of the $\mathcal{N} = 8$ one, the tensor $N_A{}^B$ makes sense as the fermion shift pertaining to the fermions $\chi^{A\alpha\beta}$ which are truncated.

If we denote by $\mathcal{V}$ the coset representative of $\mathcal{M}^{(\mathcal{N}=2)}$, the moment maps corresponding to the gauged $\mathrm{E}_{6(+2)}$ isometry reads:

$$\mathcal{P}^x \propto \theta_1{}^n\, \mathcal{V}_n{}^x \, , \tag{5.9}$$

where $\mathcal{V}_n{}^m$ is the matrix representation of $\mathcal{V}$ in the adjoint representation of $G$. The theory has an $\mathcal{N} = 2$ AdS-vacuum, corresponding to the gauging of a U(1) inside SU(2)



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and zero expectation value of the scalars in the $H$-covariant parametrization of the coset: $\langle\phi^{\alpha ABC}\rangle = 0$. Indeed such a gauging would correspond to choosing $\theta \in (\mathbf{1}, \mathbf{3})$. At the origin the T-tensor coincides with $\theta$ and thus has zero component on the $(\mathbf{20}, \mathbf{2})$ representation, implying that $(N_\alpha{}^{ABC})_{|vac.} = 0$. This gauging corresponds to a truncation of the SO(8) gauging of the $\mathcal{N} = 8$ theory. The corresponding theory cannot have an $\mathcal{N} = 2 \to \mathcal{N} = 1$ spontaneous supersymmetry breaking since it is not coupled to vector multiplets. If we gauge a U(1) subgroup of SU(6), $\theta \equiv (\theta_A{}^B) \in (\mathbf{35}, \mathbf{1})$. At the origin we would have $(N_\alpha{}^{ABC})_{|vac.} = (S^{\alpha\beta})_{|vac.} = 0$ which corresponds to an $\mathcal{N} = 2$ Minkowski vacuum, in which, depending on the eigenvalues of the $U(1)$ generator $\theta_A{}^B$, a number of hypermultiplets will become massive.


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## A Supergroups with zero Killing-Cartan form

We recall the supergroups with zero Killing-Cartan form. There are three examples

1. The first example is based on the superalgebra $A(n|n)$ with $n \geq 1$. The even part of $A(n|n)$ is $A_n \oplus A_n$ and the odd part is $(n, \bar{n}) \oplus (\bar{n}, n)$ where $A_n$ is the usual classical Lie algebra. The classical real form of this example is $\mathfrak{psu}(n|n)$ which have subalgebra $\mathfrak{su}(n) \oplus \mathfrak{su}(n)$, it is generated by supermatrices $2n \times 2n$ with vanishing supertrace and defined modulo the identity matrix $\mathbf{1}_{2n\times 2n}$ which has vanishing supertrace. The superalgebra has $\left(2n^2 - 2\middle|2\,n^2\right)$ generators, (it can be shown the corresponding supergroup manifold has vanishing Ricci curvature).

2. The second example is based on the superalgebra $D(n+1|n)$ with $n \geq 1$. The even part is $D_{n+1} \oplus C_n$ and the odd part is $(2n+2, 2n)$ where $D_n$ and $C_n$ are the classical Lie algebra series. The real form is $\mathfrak{osp}(2n+2|2n)$ (with $n \geq 1$) which has the subalgebra $\mathfrak{so}(2n+2) \times \mathfrak{sp}(2n)$. It is generated by orthosymplectic supermatrices $4n+2 \times 4n+2$. The total number of generators is $\left(4n^2 + 4n + 1\middle|4n^2 + 4n\right)$, (it can be shown the corresponding supergroup manifold has vanishing Ricci curvature).

3. The third example is based on the superalgebra $D(2, 1; \alpha)$ with $\alpha \notin \{0, -1\}$. The even part is $A_1 \oplus A_1 \oplus A_1$ and the odd part is $(2, 2, 2)$. The classical real form has bosonic subalgebra $\mathfrak{sl}(2) \oplus \mathfrak{sl}(2) \oplus \mathfrak{sl}(2)$. The total number of generators is $(9|8)$.

There are super-cosets with zero Killing forms. They are generated by symmetric cosets. Here there are some examples:

1. PSU(2,2|4)/ SO(1,4) × SO(5). This coset has 10 bosonic generators and 32 fermionic generators. The bosonic subgroup is SO(2,4) × SO(6)/ SO(1,4) × SO(5) which is $AdS_5 \times S^5$. The fermionic generators are associated to the Killing spinors of the background.

2. OSp(6|4)/ U(3) × SO(1,3). This coset has 10 bosonic generators and 24 fermions. The bosonic subgroup is SO(6) × Sp(4)/ U(3) × SO(1,3) which is $AdS_4 \times \mathbb{C}P^3$. The fermionic generators are associated to the Killing spinors which are 24.

3. $\mathrm{PSU}(n+1|n+1)/\,\mathrm{SU}(n|n+1)$ is also denoted by $\mathbb{C}P^{n|n+1}$ which is a Ricci flat supermanifold. In the case $n=3$ this is the famous Witten's supertwistor space $\mathbb{C}P^{3|4}$.

4. $\mathrm{OSp}(2n+2|2n)/\,\mathrm{OSp}(2n+1|2n)$ is also denoted by $\mathbb{S}^{2n-1|2n}$ which is the supersphere.

## B Relevant branchings and decompositions

**$\mathbf{SO^*(12) \times SU(2) \subset E_{7(7)}}$.**

$$\mathbf{56} \to (\mathbf{12},\mathbf{2}) + (\mathbf{32},\mathbf{1})\,, \tag{B.1}$$
$$\mathbf{133} \to (\mathbf{1},\mathbf{3}) + (\mathbf{66},\mathbf{1}) + (\mathbf{32'},\mathbf{2})\,, \tag{B.2}$$
$$\mathbf{912} \to (\mathbf{12},\mathbf{2}) + (\mathbf{220},\mathbf{2}) + (\mathbf{32},\mathbf{3}) + (\mathbf{352},\mathbf{1})\,. \tag{B.3}$$

**$\mathbf{SO^*(12)}$ tensor product decompositions.**

$$\mathbf{32} \times \mathbf{66} \to \mathbf{32} + \mathbf{1728} + \mathbf{352}\,, \tag{B.4}$$
$$(\mathbf{32} \times \mathbf{32} \times \mathbf{32})_s \to \mathbf{32} + \mathbf{4224} + \mathbf{1728}\,. \tag{B.5}$$

**$\mathbf{SU(6) \times SU(2) \times U(1) \subset SU(8)}$.**

$$\begin{aligned}
\mathbf{8} &\to (\mathbf{6},\mathbf{1})_{+\frac{1}{2}} + (\mathbf{1},\mathbf{2})_{-\frac{3}{2}}\,,\\
\mathbf{28} &\to (\mathbf{15},\mathbf{1})_{+1} + (\mathbf{1},\mathbf{1})_{-3} + (\mathbf{6},\mathbf{2})_{-1}\,,\\
\mathbf{36} &\to (\mathbf{21},\mathbf{1})_{+1} + (\mathbf{6},\mathbf{2})_{-1} + (\mathbf{1},\mathbf{3})_{-3}\,,\\
\mathbf{56} &\to (\mathbf{20},\mathbf{1})_{+\frac{3}{2}} + (\mathbf{6},\mathbf{1})_{-\frac{5}{2}} + (\mathbf{15},\mathbf{2})_{-\frac{1}{2}}\,,\\
\mathbf{70} &\to (\mathbf{15},\mathbf{1})_{-2} + (\overline{\mathbf{15}},\mathbf{1})_{+2} + (\mathbf{20},\mathbf{2})_{0}\,,\\
\mathbf{420} &\to (\mathbf{105},\mathbf{1})_{+1} + (\mathbf{20},\mathbf{2})_{+3} + (\mathbf{84},\mathbf{2})_{-1} + (\mathbf{15},\mathbf{1})_{+1} + (\mathbf{15},\mathbf{3})_{+1} + (\mathbf{35},\mathbf{1})_{-3} + (\mathbf{6},\mathbf{2})_{-1}\,.
\end{aligned} \tag{B.6}$$

**$\mathbf{SU(6) \times U(1) \subset SO^*(12)}$.**

$$\mathbf{12} \to \mathbf{6}_{-1} + \overline{\mathbf{6}}_{+1}\,, \tag{B.7}$$
$$\mathbf{32} \to \mathbf{1}_{+3} + \mathbf{15}_{+1} + \overline{\mathbf{15}}_{-1} + \mathbf{1}_{-3}\,, \tag{B.8}$$
$$\mathbf{32'} \to \mathbf{6}_{+2} + \mathbf{20}_{0} + \overline{\mathbf{6}}_{-2}\,, \tag{B.9}$$
$$\mathbf{66} \to (\mathbf{35}+\mathbf{1})_0 + \overline{\mathbf{15}}_{+2} + \mathbf{15}_{-2}\,, \tag{B.10}$$
$$\mathbf{352} \to \mathbf{35}_{+3} + (\mathbf{21}+\mathbf{15}+\mathbf{105})_{+1} + (\overline{\mathbf{21}}+\overline{\mathbf{15}}+\overline{\mathbf{105}})_{-1} + \overline{\mathbf{35}}_{-3}\,, \tag{B.11}$$

**SU(4) × U(1) ⊂ SO(8).** There are three inequivalent SU(4) subgroups of SO(8) which we shall denote here by $\mathrm{SU}(4)_i$, where $i = v, c, s$. With respect to $\mathrm{U}(4)_i$ the following branchings hold ($i \neq j \neq k \neq i$)

$$\begin{aligned}
\mathbf{8}_i &\to \mathbf{1}_{+1} + \mathbf{1}_{-1} + \mathbf{6}_0 \,,\\
\mathbf{8}_j &\to \mathbf{4}_{+\frac{1}{2}} + \overline{\mathbf{4}}_{-\frac{1}{2}}\\
\mathbf{8}_k &\to \mathbf{4}_{-\frac{1}{2}} + \overline{\mathbf{4}}_{+\frac{1}{2}} \,,\\
\mathbf{35}_i &\to \mathbf{1}_{+2} + \mathbf{1}_0 + \mathbf{1}_{-2} + \mathbf{6}_{+1} + \mathbf{6}_{-1} + \mathbf{20}_0 \,,\\
\mathbf{35}_j &\to \mathbf{10}_{+1} + \overline{\mathbf{10}}_{-1} + \mathbf{15}_0 \,,\\
\mathbf{35}_k &\to \mathbf{10}_{-1} + \overline{\mathbf{10}}_{+1} + \mathbf{15}_0 \,,\\
\mathbf{350} &\to \overline{\mathbf{10}}_{+1} + \overline{\mathbf{10}}_{-1} + \mathbf{10}_{+1} + \mathbf{10}_{-1} + \mathbf{6}_{+1} + \mathbf{6}_{-1} + \mathbf{45}_0 + \overline{\mathbf{45}}_0 + \mathbf{64}_{+1} + \mathbf{64}_{-1} + \mathbf{15}_{+2} +\\
&\quad + \mathbf{15}_{-2} + 2 \times \mathbf{15}_0 + \mathbf{20}'_0 \,,
\end{aligned} \tag{B.12}$$

the symmetry group associated with the $\mathcal{N} = 6$ vacuum is $\mathrm{SU}(4)_s \times \mathrm{U}(1)$, while that associated with the $\mathcal{N} = 0$ one discussed in section 4.4 is $\mathrm{SU}(4)_c \times \mathrm{U}(1)$.

## C Fermion mass terms

In this appendix we write the spin-1/2 mass terms for the the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ truncations of the $\mathcal{N} = 8$ theory. The spin-1/2 mass term for the $\mathcal{N} = 8$ theory reads

$$g\, M^{ijk,\,lmn}\, \overline{\chi}_{ijk} \chi_{lmn} \,, \tag{C.1}$$

where the mass matrix is expressed uniquely in terms of $N_i{}^{jkl}$ as follows [21]:

$$M^{ijk,\,lmn} = -\frac{1}{144}\, \epsilon^{ijkpqr[lm}\, N^{n]}{}_{pqr} \,. \tag{C.2}$$

The above equation allows us to decompose (C.1) in terms of the SU(6) × SU(2) × U(1)-irreducible tensors, introduced in section 4, and the spin-1/2 fields pertaining to the $\mathcal{N} = 6$ and $\mathcal{N} = 2$ truncations. The $\mathcal{N} = 6$ and $\mathcal{N} = 2$ spin-1/2 mass terms read:

$\mathcal{N} = 6$ :

$$-\frac{g}{24} \epsilon^{A_1 A_2 A_3 C B_1 B_2} N^{B_3}{}_C \overline{\chi}_{A_1 A_2 A_3} \chi_{B_1 B_2 B_3} \tag{C.3}$$

$$-\frac{g}{12} \epsilon^{B A_1 A_2 A_3 B_1 B_2}\, \overset{\circ}{N}{}^{B_3}{}_{A_1 A_2 A_3} \overline{\chi}_{B_1 B_2 B_3} \chi_B + \frac{g}{16} \epsilon^{A_1 A_2 A_3 B_1 B_2 B} N^{B_1 B_2} \overline{\chi}_{A_1 A_2 A_3} \chi_B \,,$$

$\mathcal{N} = 2$ :

$$\frac{g}{24} \epsilon^{\alpha\beta} \epsilon^{ABEFGC}\, \overset{\circ}{N}{}^{D}{}_{EFG} \overline{\chi}_{\alpha AB} \chi_{\beta CD} - \frac{g}{16} \epsilon^{ABCDEF} \epsilon^{\alpha\gamma}\, \overset{\circ}{N}{}^{\beta}{}_{\gamma EF} \overline{\chi}_{\alpha AB} \chi_{\beta CD} \,, \tag{C.4}$$

where the irreducible tensors $\overset{\circ}{N}_D{}^{ABC}$, $N^{AB}$, $\overset{\circ}{N}_\beta{}^{\alpha AB}$ were defined in eq. (4.26).

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